Basic Gear Design

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Description: Gears are wheel-like machine elements that have teeth uniformly spaced around the outer surface. Gears can be a fraction of an inch in diameter to a hundred feet in diameter. Gears are used in pairs and are a very valuable design tool. They are used in everything from clocks to rockets and have been around for 3000 years.

Gears are mounted on rotatable shafts and the teeth are made to mesh with a gear on another shaft. Gears deliver force (torque) and motion (rpm) from one part of a machine to another. A gearset with one gear twice size of the other will rotate at one-half the speed of the other and deliver twice the torque. Being able to control speed and torque by varying the number of teeth of one gear with respect to another makes gears a valuable design tool. An automobile transmission is an excellent example of how this principle is put to use to control vehicle motion.

Types: There are a number of different types of gears. Spur gears are the most common and the easiest to manufacture. A spur gear has teeth that are uniformly spaced around the outer surface. The teeth are aligned in a direction that is parallel to the gear axis of rotation. A spur gear is designed to mesh with another spur gear on a parallel shaft.

The profile of the contact surface of spur gear teeth is in the form of an involute curve. An involute curve is the path that the end of a string takes when it is being unwound from a cylinder. The shape is easiest to manufacture and is an efficient way to transmit power between two contacting surfaces because of the tendency to maximize rolling and minimize sliding. The efficiency of spur gears is in the high 90% range and approaches that of anti-friction bearings.

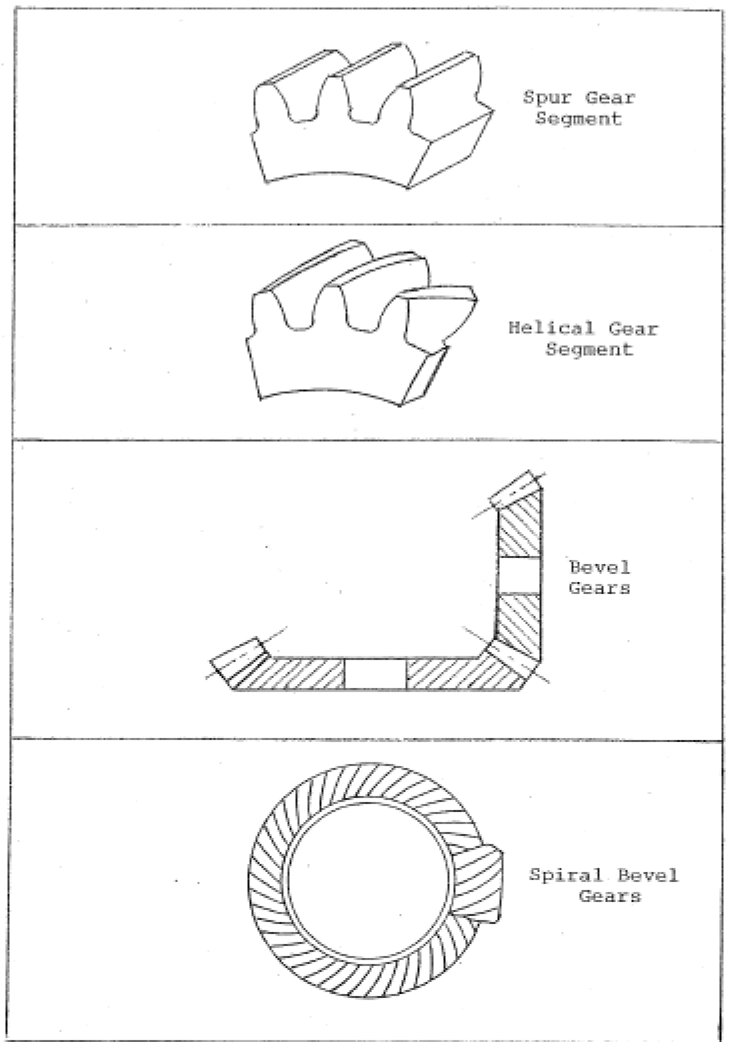
A spur gear is designed to mesh with another spur gear on a parallel shaft. Spur gears impose only radial (perpendicular) loads on shaft support bearings as opposed to other types of gears which impose radial and thrust (axial) loads on bearings. Axial loads on shafts require bearings that can support thrust as well as radial loads.

Helical gears are like spur gears except that the teeth run at an angle to the gear axis of rotation. This angle, called the helix angle, normally runs from 10° to 35°. Helical gears are stronger and run more quietly than comparable size spur gears. Helical gears are less efficient than spur gears and impose thrust as well as radial loads on shafts, making bearing selection more critical. Double helical or “herringbone” gears can deliver high torque without imposing thrust loads on shafts.

Bevel gears are like spur gears except that the basic configuration is conical shaped. This results in teeth being smaller at one end than at the other. Bevel gears are used to transmit motion between two shafts that are not parallel. Two shafts that are at a 90 degree angle are commonly connected by bevel gears. Bevel gears, like spur gears, operate at efficiencies in the high 90 percent range. Bevel gears are used in vehicles, aircraft, and machine tools. Spiral bevel gears have teeth that are cut at an angle similar to what a helical gear is to a spur gear. Spiral bevel gears operate at lower efficiencies than bevel gears. Spiral bevel gears whose axes do not intersect are called hypoid gears. See Figure 1.

Figure 1

Gear Types



Terms: the following are terms associated with gears:

Pinion is the smaller of two gears in mesh. The larger is called the gear regardless of which one is doing the driving.

Ratio is the number of teeth on the gear divided by the number of teeth on the pinion.

Pitch diameter is a basic diameter of the gear and pinion which when divided by each other equals the ratio.

Diametral pitch is a measure of tooth size and equals the number of teeth on a gear divided by the pitch diameter. Diametral pitches can range from ½ to 200.

Module is a measure of tooth size in the metric system. It equals the pitch diameter in millimeters divided by the number of teeth on a gear. Modules can range from 0.2 to 50.

Pitch circle is the circumference of the pitch diameter.

Circular pitch is the distance along the pitch circle from a point on one gear tooth to a like point on an adjacent tooth.

Addendum of a tooth is its radial height above the pitch circle. The addendum of a standard proportion tooth equals 1.000 divided by the diametral pitch. The addendum for a pinion and mating gear are equal except in the long addendum design where the pinion addendum is increased while the gear addendum is decreased by the same amount.

Dedendum of a tooth is its radial height below the pitch circle. The dedendum of a standard proportion tooth equals 1.250 divided by the diametral pitch. The dedendum for a pinion and mating gear are equal except in the long addendum design where the pinion dedendum may be decreased while the gear dedendum is increased by the same amount.

Whole depth or total height of a gear tooth equals the addendum plus the dedendum. The whole depth equals 2.250 divided by the diametral pitch.

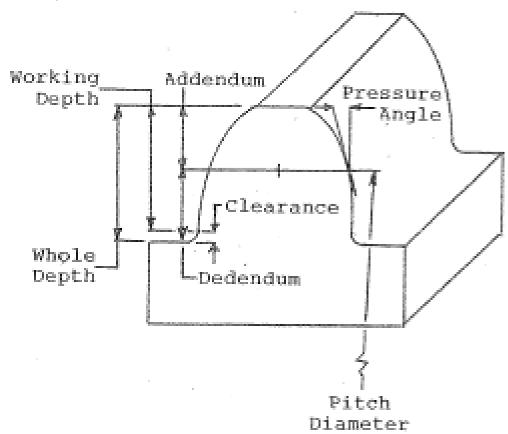
Working depth of a tooth equals the whole depth minus the radius at the base of the tooth. The working depth equals 2.000 divided by the diametral pitch.

Clearance equals the whole depth minus the working depth. The clearance is equal to the height of the radius at the base of the tooth.

Pressure angle is the slope of the gear tooth at the pitch circle. See Figure 2.

Figure 2

Gear Tooth Terminology



Dimensions For a 2 Diametral Pitch Tooth:

Addendum = 1.00/2 = 0.500 inches
Dedendum = 1.25/2 = 0.625 inches
Clearance = 0.25/2 = 0.125 inches
Whole Depth = 2.25/2 = 1.125 inches
Working Depth = 2.00/2 = 1.000 inches

Material: Gears are made out of steel, iron, bronze, and plastic. Steel is the most widely used gear material. Iron is good because of its castability and wear characteristics. Bronze is good for gears where friction is a concern. Plastic gears have good moldability properties but have limited load carrying capacity.

Many different kinds of steel can be used for gears. They range from low carbon, low alloy to high carbon, high alloy grades. The type used depends on load, size and cost considerations. Low carbon, low alloy steels are used when low cost is of prime importance. High carbon, high alloy grades are used when small size and high load are the major design objectives.

Steel gears can be heat treated to improve performance by increasing strength and wear properties. Some alloys are thru-hardened to the Rockwell C42 level. Others are carburized and hardened to the Rockwell C60 level on an outer shell leaving the inner core softer. This hardening technique called case-hardening imparts good strength and wear properties to the outer layer while the softer inner core gives good shock absorbing characteristics.

Gear steels come in grades 1, 2, and 3. Higher grade numbers represent higher quality steels which are used for higher performing gears. The items controlled are material composition, residual stress and microstructure.

Manufacture: gear cutting processes can be classified as either generating or forming. The generating method involves moving the tool over the work piece in such a way as to create the desired shape. In the forming process, the shape of the tool is imparted on the work piece.

A generating method of cutting gear teeth that is commonly used is called hobbing. A hob is a thread-like tool with a series of slots machined across it to provide cutting surfaces. The tool is fed across the gear blank developing several teeth at the same time. Forming methods of gear cutting include shaping and milling. Shaping uses a gear like tool that is reciprocated up and down to impart its tooth form to the gear blank. Milling uses a shaped tool to remove the material between gear teeth.

After cutting, some gears are heat treated to increase strength and wearability. This process causes a small amount of distortion. In order to restore good tooth accuracy and surface finish, heat treating is followed by a finishing operation. For gears that are heat treated to a hardness below

Rockwell C42, a finishing operation called shaving is performed. Shaving is similar to shaping except that the tool teeth are grooved to provide additional cutting edges to remove a small amount of material. For gears that are heat treated to a hardness of Rockwell C42 or higher, grinding is used as the finishing operation. Grinding can either be a generating or forming method of finishing gears. The generating method passes an abrasive wheel over the gear teeth in a prescribed manner to true up the teeth and produce a fine surface finish. The forming method feeds a shaped wheel between the gear teeth similar to milling.

Gears can be manufactured over a very wide size range. They can be from a fraction of an inch in diameter to many feet in diameter. Gear tooth height can range from .001 inch (200 diametral pitch) to 4.31 inches (½ diametral pitch). Metric tooth height ranges from .431 millimeters (0.2 module) to 107.85 millimeters (50 module).

Gear teeth are normally manufactured with pressure angles ranging from 14.5° to 25°. As the pressure angle is made larger, the teeth become wider at the base and narrower at the tip. This makes the teeth stronger and able to carry more load but more apt to chipping at the tip if not designed properly. Pinions can be made with fewer teeth with higher pressure angles because of there being less danger of undercutting. Undercutting is a narrowing of the bottom of the teeth when being manufactured. Lower pressure angle teeth have a narrower base and carry less load than higher pressure angle teeth but the teeth are wider at the tip and less apt to chipping. Finally, lower pressure angle teeth run more smoothly and quieter than higher pressure angle teeth because of having higher contact ratios. Contact ratio is a measure of the number of teeth in engagement at the same time during gear operation.

Spur Gear Design: As an exercise in gear design, a step-by-step procedure will be demonstrated on how to design a spur gearset (pinion and gear) for a general industrial gearbox. The gearset shall transmit 100 horsepower at a pinion speed of 1000 revolutions per minute. It shall have a ratio of 8 to 1 and use standard proportion 25 degree pressure angle teeth.

Initially, a determination must be made as to the number of teeth in the pinion. The goal is to keep the number of teeth as low as possible to minimize weight and cost while still fulfilling all the design objectives. From the American Gear Manufacturers Association Information Sheet AGMA 908-B89 (AGMA 1), the fewest number of standard proportion 25 degree pressure angle teeth that a pinion can have is 14. Fewer than 14 will cause undercutting. Undercutting, as previously mentioned, is a narrowing or weakening at the base of the gear teeth. It is a characteristic of gear manufacturing caused by trying to machine a gear with too few teeth.

Through-hardened Rockwell C42 steel and case-hardened Rockwell C60 steels will be investigated for the gearset. As a general rule of thumb for steels in this hardness range, the number of teeth in the pinion should be from 30 for low ratio (1/1) gearsets to 14 for high ratio (10/1) gearsets. As a result, a pinion with 17 teeth will be selected for the 8 to 1 ratio in the example.

An 8 to 1 gearset ratio requires the gear to have 8x17=136 teeth. A tooth ratio of 136 to 17 is not a "hunting ratio". Hunting ratios occur when each pinion tooth contacts every gear tooth before it contacts any gear tooth a second time. Hunting ratios tend to equalize tooth wear and improve tooth spacing. The test for a hunting ratio is that the number of teeth in the pinion and the number of teeth in the gear, cannot be divided by the same number excluding 1. Both tooth numbers in a 136 to 17 ratio can be divided by 17. In order to make the ratio a hunting ratio, a tooth will be dropped from the gear and a hunting ratio of 135 to 17 will be used. The new ratio of 7.9 to 1 is close enough to 8 to 1 for this application.

Spur Gear Rating: There are a number of methods that are used to rate spur gears. The American Gear manufacturers Association offers two ways to do it. Both are contained in AGMA Standard ANSI/AGMA 2001-D04 (AGMA 2) and AGMA Information Sheet AGMA 908-B89 (AGMA 1). One way calculates the allowable transmitted horsepower on the pitting resistance of gear teeth contact surfaces while the other calculates transmitted horsepower on gear teeth bending strength.

The two power rating formulas will be used to evaluate the gearset with two different materials and four different diametral pitches (tooth sizes) in order to optimize the design. The first material will be steel, thru-hardened to Rockwell C42, while the second will be steel, case-hardened to Rockwell C60. All metallurgical properties of the pinion and gear will be assumed to be grade 1 specification. Pinion and gear as finished surface topography are assumed to be commensurate with good gear manufacturing practice. The diametral pitches to be investigated shall be 7.00, 6.77, 6.35, and 6.00. The 7.00 and 6.00 diametral pitches are standard English sizes while the 6.77 (4 module) and the 6.35 (3.75 module) are standard metric sizes. The two materials and the diametral pitches were selected based on preliminary analysis using the AGMA formulas (AGMA 1,2).

The pitting resistance power rating formula transposed down to one line is as follows:

$$P_{ac}= (\pi\, n_p F/396{,}000)\ (I/K_o K_v K_s K_m C_f)\ [(ds_{ac}/C_p S_H)\ (Z_N C_H/K_T K_R)]^{2}$$

P_{ac} is the pitting resistance allowable transmitted horsepower for 10 million cycles of operation at 99 percent reliability.

π is a constant and equals 3.142.

n_p is the pinion speed which equals 1000 rpm.

F is the face width of the gears. As a general rule of thumb for industrial gears, F=1.0xd where d is the pinion pitch diameter. F and d are calculated below.

I is the tooth geometry factor for pitting resistance which from AGMA 908-B89 (AGMA 1) equals .132 for 25° pressure angle standard proportion teeth and a ratio of 135/7.

K_o, K_v, K_s, K_m, and C_f are rating factors for overload, dynamics, size, load distribution, and surface condition, respectively. For the sake of brevity, they will be assumed to be 1. In actual practice they all must be evaluated per ANSI/AGMA 2001-D04 (AGMA 2).

d is the pinion pitch diameter and for the four diametral pitches being investigated, equals 17/7=2.429”, 17/6.77=2.511”, 17/6.35=2.677”, and 17/6.00=2.833”. Since the face width F equals 1.0xd, the face widths and pitch diameters are the same value.

s_{ac} is the allowable contact stress and from ANSI/AGMA 2001-D04 (AGMA 2) equals 158,000 psi for grade 1, RC42 through-hardened steel, and 180,000 psi for grade 1, RC60 case-hardened steel.

C_p is the elastic coefficient and from ANSI/AGMA 2001-D04 (AGMA 2) equals 2300 psi for a steel pinion and gear.

S_H, Z_N, C_H, K_T, and K_R are rating factors for safety, stress cycle, hardness ratio, temperature, and reliability, respectively. For the sake of brevity, they will be assumed to be 1. In actual practice, they all must be evaluated per ANSI/AGMA 2001-D04. (AGMA 2)

The allowable pitting resistance transmitted horsepowers for the two steels and four diametral pitches are as follows:

1) Thru-hardened RC42 steel and 7.00 diametral pitch:
$(3.142x1000x2.429x.132/396,000)(2.429x158,000/2300)^2= 70.9hp$

2) Case-hardened RC60 steel and 7.00 diametral pitch:
$(3.142x1000x2.429x.132/396,000)(2.429x180,000/2300)^2= 92.0hp$

3) Thru-hardened RC42 steel and 6.77 diametral pitch:
$(3.142x1000x2.511x.132/396,000)(2.511x158,000/2300)^2= 78.1hp$

4) Case-hardened RC60 steel and 6.77 diametral pitch:
$(3.142x1000x2.511x.132/396,000)(2.511x180,000/2300)^2= 101.4hp$

5) Thru-hardened RC42 steel and 6.35 diametral pitch:
$(3.142x1000x2.677x.132/396,000)(2.677x158,000/2300)^2= 95.1hp$

6) Case-hardened RC60 steel and 6.35 diametral pitch:
$(3.142x1000x2.677x.132/396,000)(2.677x180,000/2300)^2= 123.4hp$

7) Thru-hardened RC42 steel and 6.00 diametral pitch:
$(3.142x1000x2.833x.132/396,000)(2.833x158,000/2300)^2= 112.0hp$

8) Case-hardened RC60 steel and 6.00 diametral pitch:

$(3.142x1000x2.833x.132/396,000)(2.833x180,000/2300)^2 = 145.3hp$

The bending strength power rating formula transposed down to one line is as follows:

$$P_{at} = (\pi n_p d/396,000K_oK_v)(F/P_d)(J/K_sK_mK_B)(s_{at}Y_N/S_FK_TK_R)$$

P_{at} is the bending strength allowable transmitted horsepower for 10 million cycles of operation at 99% reliability.

π is a constant and equals 3.142.

n_p is the pinion speed which equals 1000 rpm.

d is the pinion pitch diameter and, as previously calculated, equals 2.429", 2.511", 2.677", and 2.833" for diametral pitches 7.00, 6.77, 6.35, and 6.00, respectively.

K_o and K_v are rating factors for overload and dynamics, respectively. For the sake of brevity, they will be assumed to be 1. In actual practice, they all must be evaluated per ANSI/AGMA 2001-D04. (AGMA 2)

F is the face width of the gears and equals 2.429", 2.511", 2.677", and 2.833" for the four cases being investigated.

P_d is the diametral pitch and equals 7.00, 6.77, 6.35, and 6.00 for the four cases being investigated.

J is the tooth geometry factor for bending strength which from AGMA 908-B89 (AGMA 1) equals .30 for the pinion and .38 for the gear. The .30 pinion factor will be used since it will yield the more conservative result.

K_s, K_m, and K_B are rating factors for size, load distribution, and rim thickness, respectively. For the sake of brevity, they will be assumed to be 1. In actual practice, they all must be evaluated per ANSI/AGMA 2001-D04. (AGMA 2).

S_{at} is the allowable bending stress and from ANSI/AGMA 2001-D04 (AGMA 2) is 43,700 psi for grade 1, RC42 through-hardened steel, and 55,000 psi for grade 1, RC60 case-hardened steel.

Y_N, S_F, K_T, and K_R are rating factors for stress cycles, safety factor, temperature and reliability, respectively. For the sake of brevity, they will be assumed to be 1. In actual practice, they all must be evaluated per ANSI/AGMA 2001-D04. (AGMA 2).

The allowable bending strength transmitted horsepower for the two steels and four diametral pitches are as follows:

1) Thru-hardened RC42 steel and 7.00 diametral pitch:
(3.142x1000x2.429x2.429x.30x43,700) / (396,000x7.00) = 87.7hp

2) Case-hardened RC60 steel and 7.00 diametral pitch:
(3.142x1000x2.429x2.429x.30x55,000) / (396,000x7.00) = 110.4hp

3) Thru-hardened RC42 steel and 6.77 diametral pitch:
(3.142x1000x2.511x2.511x.30x43,700) / (396,000x6.77) = 96.7hp

4) Case-hardened RC60 steel and 6.77 diametral pitch:
(3.142x1000x2.511x2.511x.30x55,000) / (396,000x6.77) = 121.8hp

5) Thru-hardened RC42 steel and 6.35 diametral pitch:
(3.142x1000x2.677x2.677x.30x43,700) / (396,000x6.35) = 117.6hp

6) Case-hardened RC60 steel and 6.35 diametral pitch:
(3.142x1000x2.677x2.677x.30x55,000) / (396,000x6.35) = 148.0hp

7) Thru-hardened RC42 steel and 6.00 diametral pitch:
(3.142x1000x2.833x2.833x.30x43,700) / (396,000x6.00) = 138.8hp

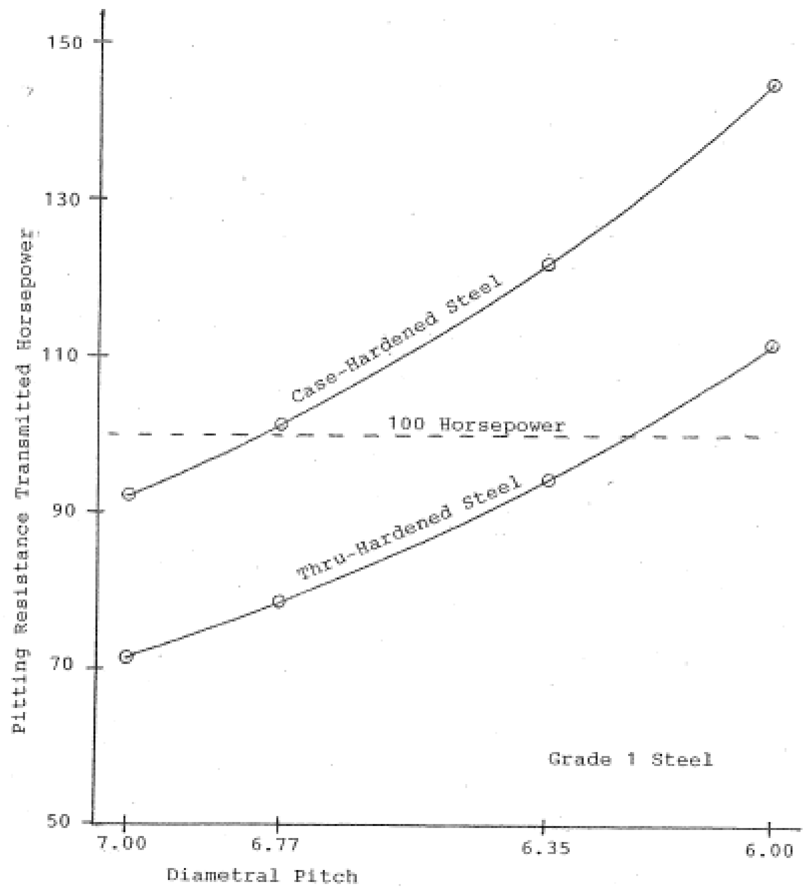
8) Case-hardened RC60 steel and 6.00 diametral pitch:
(3.142x1000x2.833x2.833x.30x55,000) / (396,000x6.00) = 174.6hp

It can be seen from the results of the pitting and bending calculations, that, when comparing like designs, the pitting transmitted horsepowers are somewhat lower than the bending transmitted horsepowers. Since this shows that the gearset is more likely to fail from pitting, the more conservative pitting numbers will be used to evaluate the various design options.

Figure 3 is a graph of pitting transmitted horsepower versus diametral pitch for both thru-hardened and case-hardened steel. It can be seen that, for the thru-hardened steel, only the 6.00 diametral pitch gearset meets the 100 horsepower design objective while, for the case-hardened steel, the 6.00, 6.35, and 6.77 diametral pitch gears all meet the 100 hp design objective. Size and weight are not as important as low cost for this general industry application; therefore, the thru-hardened 6.00 diametral pitch option would be selected because normally the case-hardened designs are more expensive. In order to be sure that the thru-hardened gearset meets all the design objectives, the pitting formula would now have to be run with all the modifying factors included.

Figure 3

Pitting Resistance Transmitted Horsepower
Versus
Diametral Pitch



Material Upgrade: In the preceding problem, only grade 1 materials were considered; however, performance of the gearset can be enhanced by using higher grade steels. Thru-hardened steel can be provided in grades 1 and 2, while case-hardened steel can be supplied in grades 1, 2, and 3. The higher grade numbers represent improved steel composition, cleanliness, and microstructure along with stricter heat treat process controls.

In order to compare the performance of the higher grade steels to the grade 1 type that was previously used, the pitting power rating formula will be repeated for the higher grade materials. The only factor that changes in the formula is the s_{ac} allowable contact stress factor. For through-hardened steel, s_{ac} is 158,000 psi for grade 1 steel and 174,000 psi for grade 2 steel. For case-hardened steel, the value of s_{ac} is 180,000 psi for grade 1 steel, 225,000 psi for grade 2 steel, and 275,000 psi for grade 3 steel. The calculations for the higher grade steels follow:

1) Thru-hardened grade 2 and 7.00 diametral pitch:
$(3.142x1000x2.429x.132/396,000)(2.429x174,000/2300)^2$= 86.0hp

2) Case-hardened grade 2 and 7.00 diametral pitch:
$(3.142x1000x2.429x.132/396,000)(2.429x225,000/2300)^2$= 143.7hp

3) Case-hardened grade 3 and 7.00 diametral pitch:
$(3.142x1000x2.429x.132/396,000)(2.429x275,000/2300)^2$= 214.7hp

4) Thru-hardened grade 2 steel and 6.77 diametral pitch:
$(3.142x1000x2.511x.132/396,000)(2.511x174,000/2300)^2$= 94.7hp

5) Case-hardened grade 2 and 6.77 diametral pitch:
$(3.142x1000x2.511x.132/396,000)(2.511x225,000/2300)^2$= 158.4hp

6) Case-hardened grade 3 and 6.77 diametral pitch:
$(3.142x1000x2.511x.132/396,000)(2.511x275,000/2300)^2$= 236.7hp

7) Thru-hardened grade 2 and 6.35 diametral pitch:
$(3.142x1000x2.677x.132/396,000)(2.677x174,000/2300)^2$= 115.3hp

8) Case-hardened grade 2 and 6.35 diametral pitch:
$(3.142x1000x2.677x.132/396,000)(2.677x225,000/2300)^2$= 192.8hp

9) Case-hardened grade 3 and 6.35 diametral pitch:
$(3.142x1000x2.677x.132/396{,}000)(2.677x275{,}000/2300)^2$= 288.0hp

10) Thru-hardened grade 2 and 6.00 diametral pitch:
$(3.142x1000x2.833x.132/396{,}000)(2.833x174{,}000/2300)^2$= 135.8hp

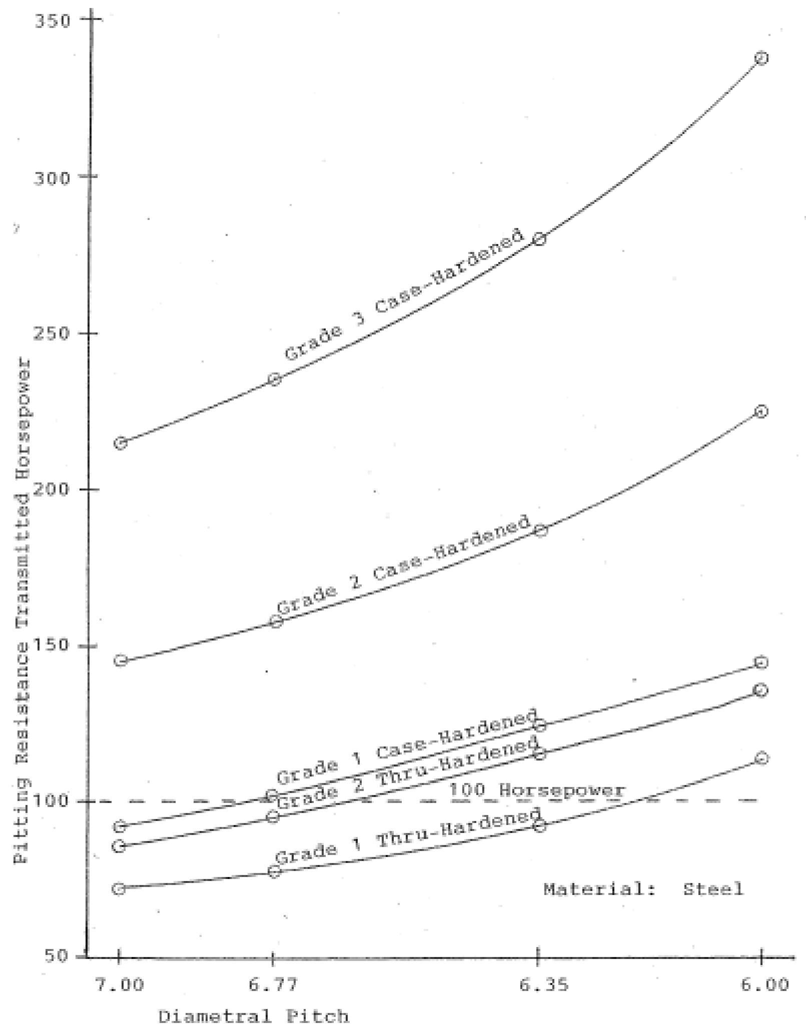
11) Case-hardened grade 2 and 6.00 diametral pitch:
$(3.142x1000x2.833x.132/396{,}000)(2.833x225{,}000/2300)^2$= 227.0hp

12) Case-hardened grade 3 and 6.00 diametral pitch:
$(3.142x1000x2.833x.132/396{,}000)(2.833x275{,}000/2300)^2$= 339.1hp

The results of the new calculations with the higher grade steels along with the previous results with grade 1 steel are shown on Figure 4. It can be seen that the higher grade level steels provide substantially higher horsepower ratings than grade 1 steel. Grade 2 thru-hardened steel rated 10% higher than grade 1 thru-hardened steel. Grade 2 case-hardened steel rated 25% higher than grade 1 case-hardened steel. Grade 3 case-hardened steel rated 53% higher than grade 1 case-hardened steel. This exercise demonstrates the important role that steel quality plays in the performance of gears.

Figure 4

Pitting Resistance Transmitted Horsepower
Versus
Diametral Pitch



Long Addendum Design: Normally, in a gearset, the pinion is weaker than the gear. In order to equalize the strength of the two, a tooth modification called "long addendum" is used. In the long addendum design, the pinion addendum is increased while the gear addendum is decreased by the same amount. This not only increases the pinion teeth bending strength, but it also reduces the stresses that cause pitting failure.

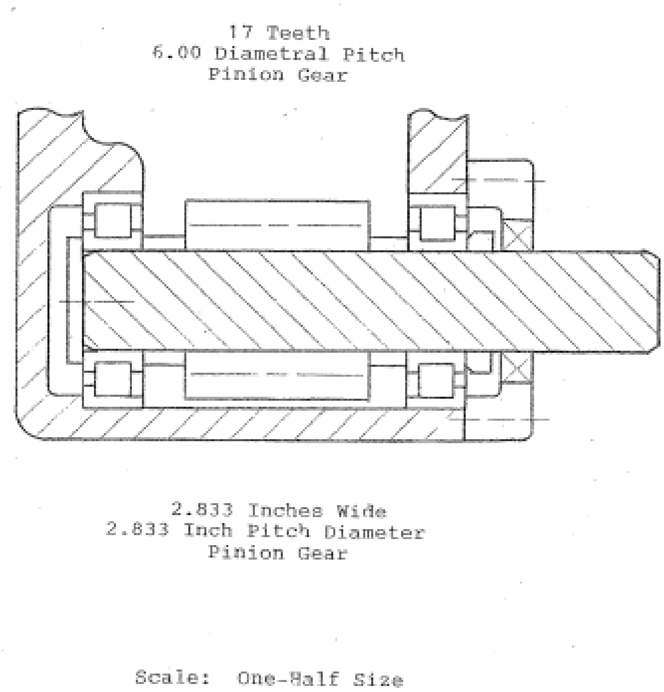
In the previous sample problems, standard length pinion and gear addenda were used. The pinion addendum will now be increased to 125% and the results compared. The same AGMA (1,2) formulas will be used. As before, the only things that change are the tooth geometry factors. When applying tooth geometry factor ratios to the already calculated horsepower ratings, the pitting resistance horsepower rating is increased by 14% while the pinion tooth bending strength horsepower rating is increased by 13%. For the helical gear version of the same gearset, the above percentages are similar. Since the long addendum tooth modification is somewhat easy to accommodate in manufacturing, it is a valuable engineering design tool to use.

Spur Gear Mounting: After a gearset has been designed, the bearings must be selected and the shafts and rims stress analyzed. The bearings selected must have the capacity to support the loads without failure over the entire design life of the gears. The shafts must be strong enough to support the gears without failure or without excessive deflection which, in itself, can cause gear and bearing failure. The gear rims must have sufficient radial thickness to prevent fatigue cracks from propagating through the rim rather than through the teeth.

As a design exercise, the bearings will be selected and the shafts and rims stress analyzed for the pinion gear in the 6.00 diametral pitch spur gearset chosen in the original problem. The pinion is the driving member. The gear is centrally located between roller bearings that are spaced 5 inches on center. Since spur gears impose no appreciable thrust load, roller bearings with their limited thrust capacity are used although ball bearings could also be used. The bearings must have an inside diameter that will fit over a shaft that is strong enough for the application. The bearing outside diameter should be larger than the gear outside diameter so the gear-shaft-bearing assembly can be easily installed and removed from the gearbox. See Figure 5.

Figure 5

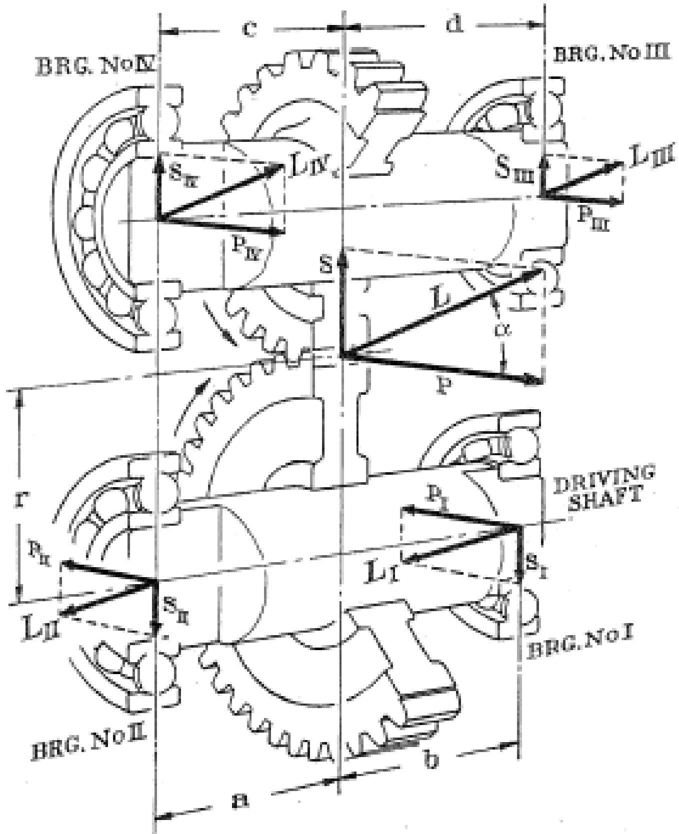
Pinion Gear Mounting



Spur Gear Bearing Selection: Bearing selection involves calculating loads, choosing a bearing, and completing a life calculation. Figure 6 has a sketch and accompanying formulas for calculating the load on the two pinion shaft bearings in question. Disregarding the relative size of the two gears shown on Figure 6, the pinion shaft bearings are number I and II. After inserting the correct values into the equations of Figure 6, the load on each of the two bearings is found to be 2455 pounds.

Figure 6

Bearing Loads Due to
STRAIGHT SPUR GEARS



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Bearing Loads Due to
STRAIGHT SPUR GEARS

Bearing Loads Due to
Tangential and Separating Forces at Tooth Contact

$Q = \dfrac{H.P. \times 63025}{N}$ = TORQUE INPUT, lbs. inches, where
H.P. = Horsepower transmitted.
N = Revolutions per minute of driving gear.

$P = \dfrac{Q}{r}$ = TANGENTIAL FORCE of driving gear, where
r = Pitch radius of gear in inches.

$S = P \tan \alpha$ = SEPARATING FORCE, where
α = Tooth pressure angle.
L = Total load on both shafts due to gears.

BEARING LOADS

Due to	on Brg. I	on Brg. II	on Brg. III	on Brg. IV
P	$P\dfrac{a}{a+b} = P_I$	$P\dfrac{b}{a+b} = P_{II}$	$P\dfrac{c}{c+d} = P_{III}$	$P\dfrac{d}{c+d} = P_{IV}$
S	$S\dfrac{a}{a+b} = S_I$	$S\dfrac{b}{a+b} = S_{II}$	$S\dfrac{c}{c+d} = S_{III}$	$S\dfrac{d}{c+d} = S_{IV}$

$L_I = \sqrt{P_I^2 + S_I^2}$ $L_{II} = \sqrt{P_{II}^2 + S_{II}^2}$ $L_{III} = \sqrt{P_{III}^2 + S_{III}^2}$ $L_{IV} = \sqrt{P_{IV}^2 + S_{IV}^2}$

Speed Change

r.p.m. of driven gear $= N \times \dfrac{\text{Number of teeth in driver}}{\text{Number of teeth in driven}}$

Now that the bearing loads are known, the following formula will be used to calculate the bearing life:

$$L = 3000(\text{bearing rating}) / (\text{bearing load})^{10/3} x(500/\text{pinion rpm})$$

L is the life of the bearing in B10 hours. B10 hours are the hours that 90 percent of the bearings are expected to run without failure.

The rating for the bearing selected is 3700 pounds and is found in industry catalogs.

The bearing load is 2455 pounds.

The pinion speed is 1000 revolutions per minute.

After inserting the various quantities, the equation now becomes:

$$L_b=3000(3700/2455)^{10/3} \text{ x } (500/1000) = 5886 \text{ B10 hours}$$

The life of the gearset, as previously mentioned, is 10 million cycles at 99 percent reliability. The life in hours is:

$$L_g = 10 \text{ x } 10^6 \text{ cycles/ } [(\text{pinion rpm}) \text{ x } 60]$$

$$L_g = 10 \text{ x } 10^6/(1000\text{x}60) = 166 \text{ hr}$$

A conservative way to convert bearing B10 hours into gear 99 percent reliability hours is to divide the bearing B10 hours by 10:

$$5886/10=588 \text{ hr}$$

The converted bearing life of 588 hours is much greater that the gear life of 166 hours indicating excellent bearing performance can be expected for this application.

Gear Shaft Design: Gear shaft design, in this case, involves making strength and deflection calculations on the straight shaft shown on Figure 5. Strength calculations are made to determine that the shaft is large enough to carry the gear loads without fracture or fatigue failure. Deflection calculations determine that shaft bending is not great enough to put excessive misalignment on the gear and bearings.

The following equation taken from The American Society of Mechanical Engineers "Code for Design of Transmission Shafting" (Mechanical Engineering) will be used to determine the minimum shaft diameter needed to support the load:

$$D = [16[(K_m M)^2 + (K_t T)^2]^{1/2} / \pi p_t]^{1/3}$$

D is the minimum outside diameter of the shaft (in inches) needed to support the prescribed bending and torsional loading.

K_m is a shock and fatigue rating factor which is 1.5 for a rotating shaft.

M is the maximum moment on the shaft and equals 3068 in-lbs. It was calculated using a simply supported beam equation.

K_t is another shock and fatigue factor which is 1.0 for a rotating shaft.

T is the transmitted torque and, from the equation on Figure 6, can be found to be 6302.5 in-lbs.

π is a constant and equals 3.142.

p_t is the allowable shear stress of the shaft material. It equals 0.18 x 63,000 or 11,340 psi for low carbon steel bar. If a keyway is present, the value of p_t must be increased by 25%.

Solving the equation:

$$D = [16[(1.5x3068^2) + (6302.5)^2]^{½} /3.142x11,340]^{1/3} = 1.518 \text{ in}$$

The equation reveals that a shaft diameter of 1.518 inches is needed to support the bending and torsional loading. The diameter of the shaft in Figure 5 is 1.575 inches which is the equivalent of the bearing 40 millimeter inside diameter. Since 1.575 inches is greater than 1.219 inches, the shaft strength is more than adequate for the application.

A maximum shaft misalignment of .0006 in/in has been calculated. It involves solving a simply supported beam equation. Since the gear is centrally located, it sees virtually no misalignment. The maximum misalignment occurs at the shaft ends under the roller bearings. Since roller bearings can tolerate a misalignment of .001 in/in maximum, the shafts are shown to be rigid enough for the application.

<u>Gear Rim Thickness</u>: The gear rim is the ring of material that lies under the gear teeth and serves to hold and support the gear teeth. The gear rim must be of sufficient radial thickness to prevent fatigue cracks from propagating through the rim rather than through the gear teeth. ANSI/AGMA 2001-D04 (AGMA 2) recommends that the gear rim thickness be no less than 1.2 times the whole tooth depth. A method is presented that downgrades bending strength power ratings for gears with insufficient rim thickness.

The rim thickness backup ratio for the pinion gear shown on Figure 5 equals .421/.375 or 1.12. Since 1.12 is less than 1.20, the ANSI/AGMA 2001-D04 (AGMA 2) formula must be applied. The formula is as follows:

$$K_B = 1.6x\ln (2.242/m_B)$$

K_B is the rim thickness factor.

m_B is the rim backup ratio which, from above, equals 1.12.

Solving the equation:

$$1.6x\ln (2.242/1.12) = 1.11$$

The rim thickness factor as applied to the previously calculated bending strength power rating equals 138.8/1.11 or 125 horsepower. The 125 horsepower is greater than the 100 horsepower that the Figure 5 gearset is being designed for; therefore, based on the above, the gearset is satisfactory for the application. However, no allowance was made for the presence of a keyway in the rim. In actual practice, such an allowance must be made. The rim thickness factor for the larger driven gear can easily be made 1.2 since the gear is much larger than the shaft.

Spur Gear Rating Study: Figures 7 through 12 contain graphs of allowable transmitted horsepower versus pressure angle, gear ratio, and steel grade for diametral pitches of 15, 10, and 5 and long pinion addenda of 0%, 25%, and 50%. Three different calculations were made at each data point using the method prescribed in ANSI/AGMA 2001-D04 (AGMA 2). Calculations were made for pitting horsepower, pinion bending horsepower, and gear bending horsepower. Of the three, only the lowest rated horsepower was plotted.

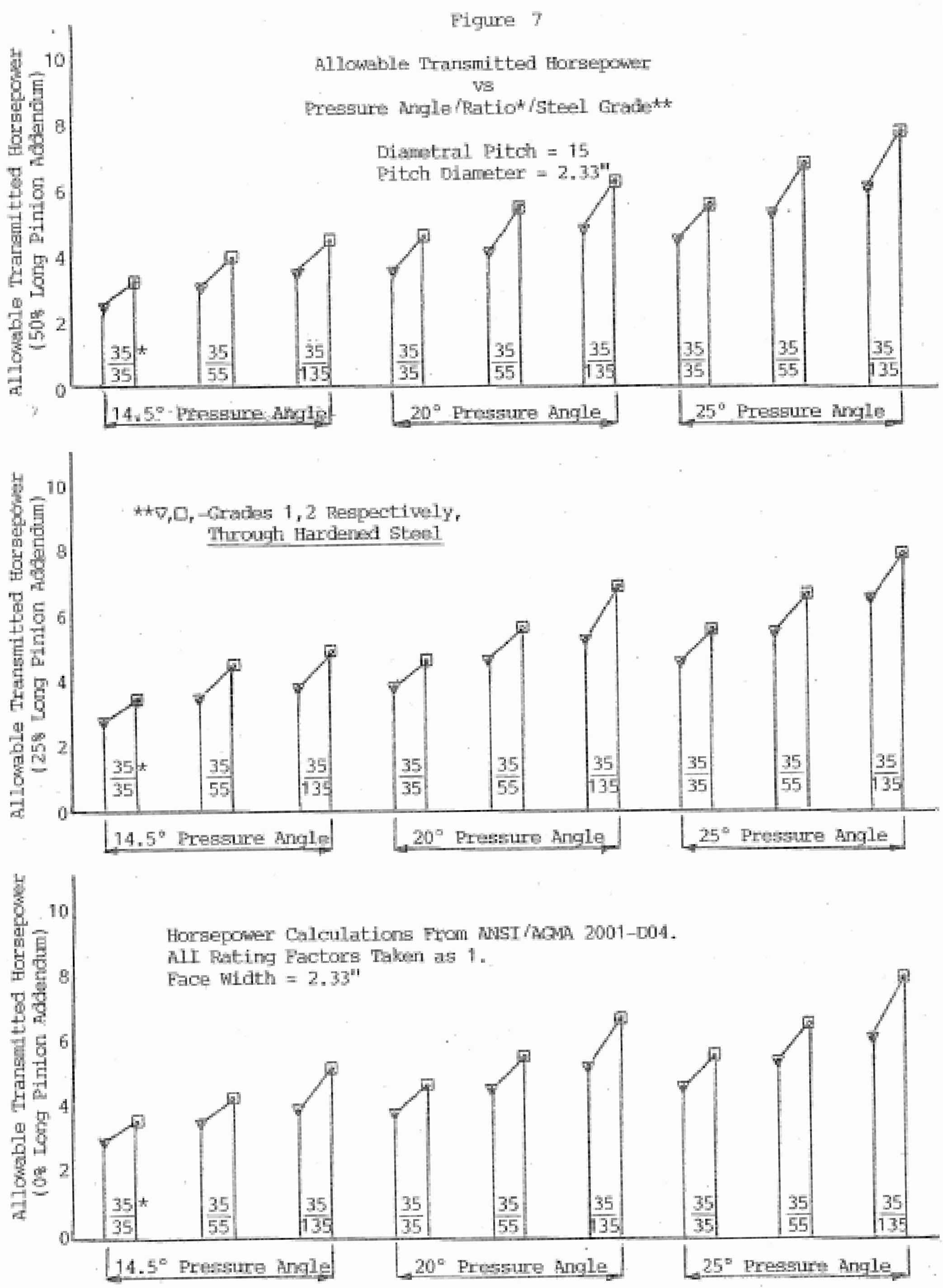


Figure 7
Allowable Transmitted Horsepower
vs
Pressure Angle/Ratio*/Steel Grade**
Diametral Pitch = 15
Pitch Diameter = 2.33"
Allowable Transmitted Horsepower (50% Long Pinion Addendum)
0
2
4
6
8
10
35/35*
35/55
35/135
35/35
35/55
35/135
35/35
35/55
35/135
14.5° Pressure Angle
20° Pressure Angle
25° Pressure Angle
**▽,□,-Grades 1,2 Respectively, Through Hardened Steel
Allowable Transmitted Horsepower (25% Long Pinion Addendum)
Horsepower Calculations From ANSI/AGMA 2001-D04.
All Rating Factors Taken as 1.
Face Width = 2.33"
Allowable Transmitted Horsepower (0% Long Pinion Addendum)

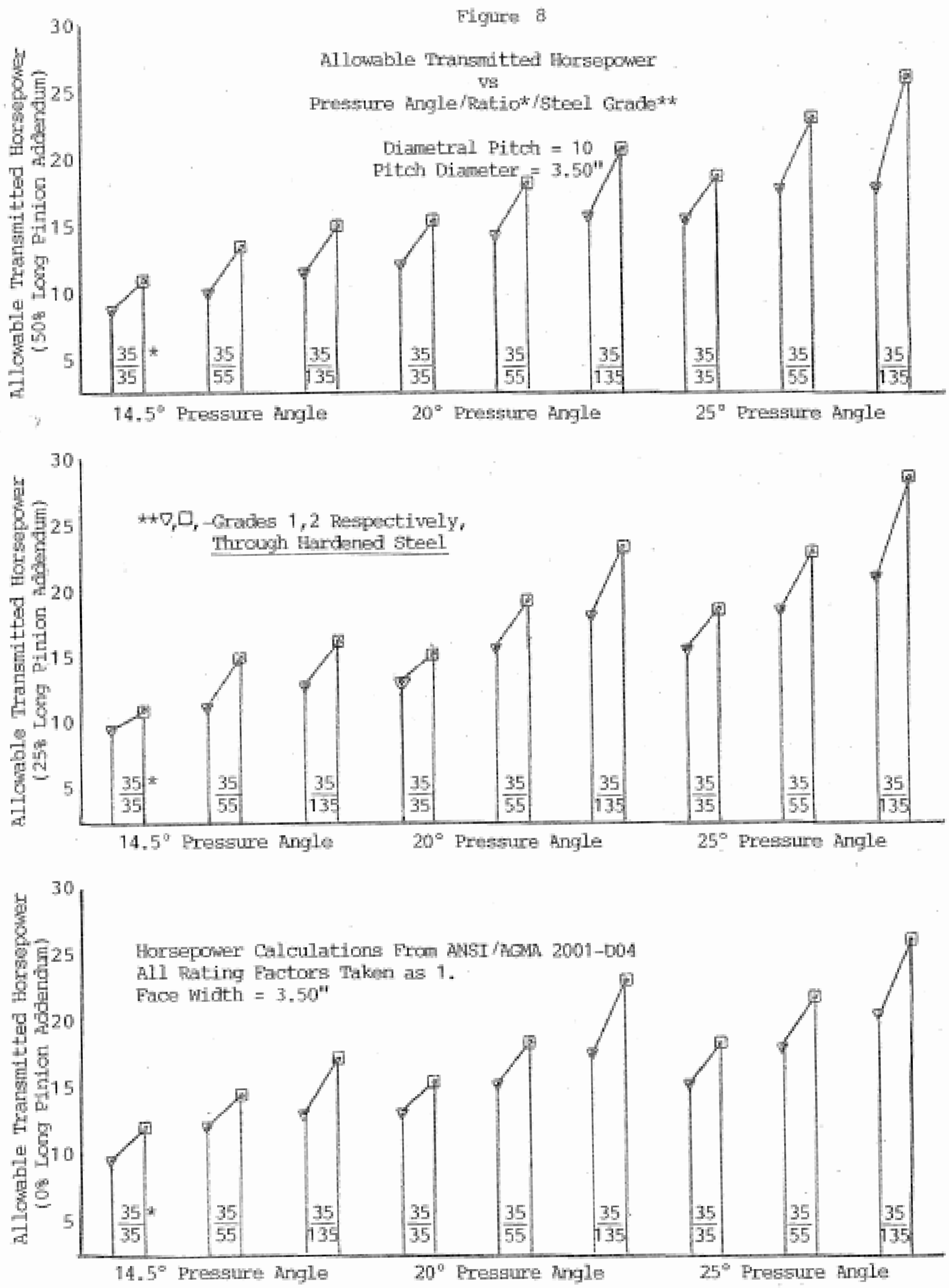


Figure 8
Allowable Transmitted Horsepower
vs
Pressure Angle/Ratio*/Steel Grade**
Diametral Pitch = 10
Pitch Diameter = 3.50"
Allowable Transmitted Horsepower (50% Long Pinion Addendum)
**▽,□,-Grades 1,2 Respectively,
Through Hardened Steel
Allowable Transmitted Horsepower (25% Long Pinion Addendum)
Horsepower Calculations From ANSI/AGMA 2001-D04
All Rating Factors Taken as 1.
Face Width = 3.50"
Allowable Transmitted Horsepower (0% Long Pinion Addendum)
35/35 *
35/55
35/135
14.5° Pressure Angle
20° Pressure Angle
25° Pressure Angle

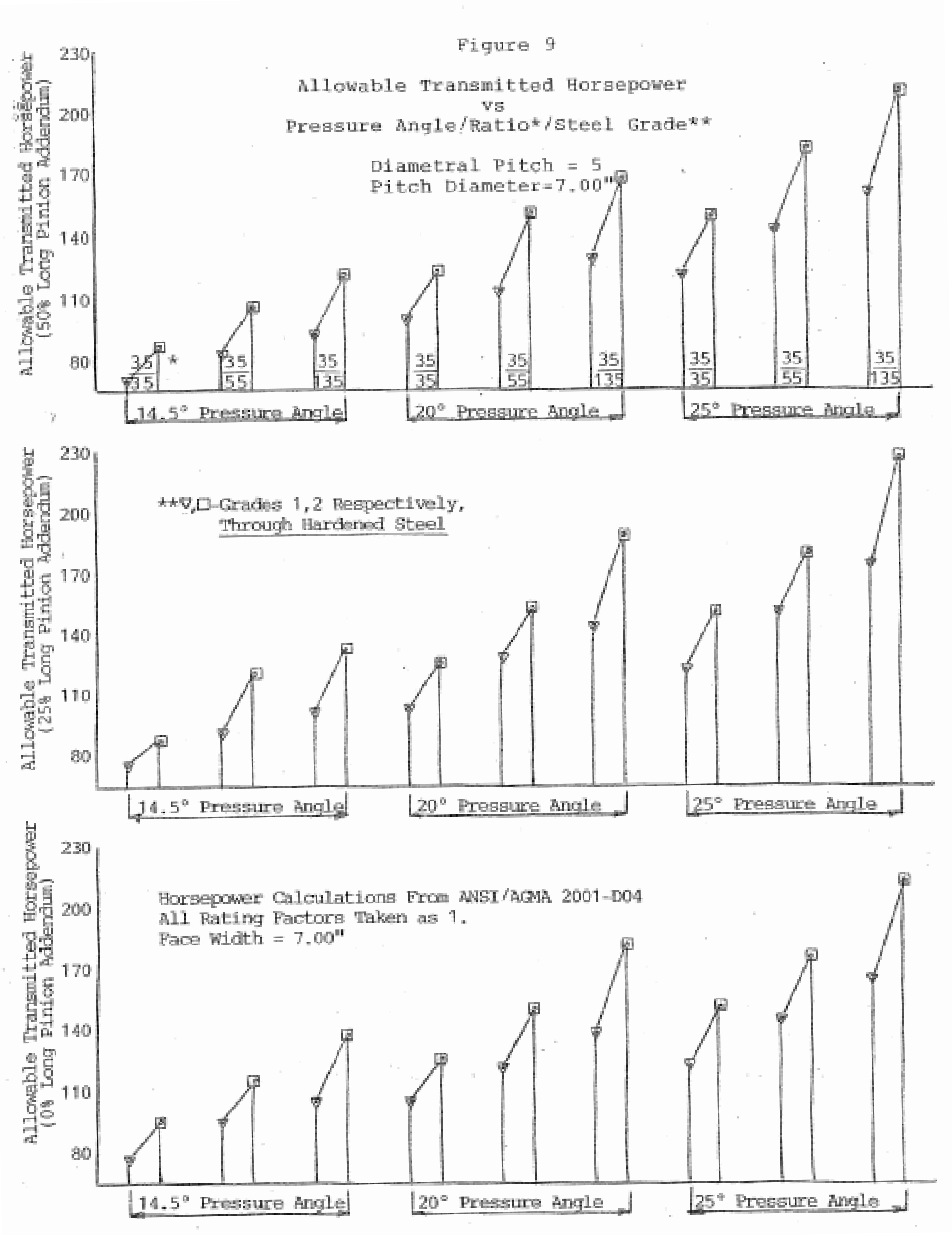
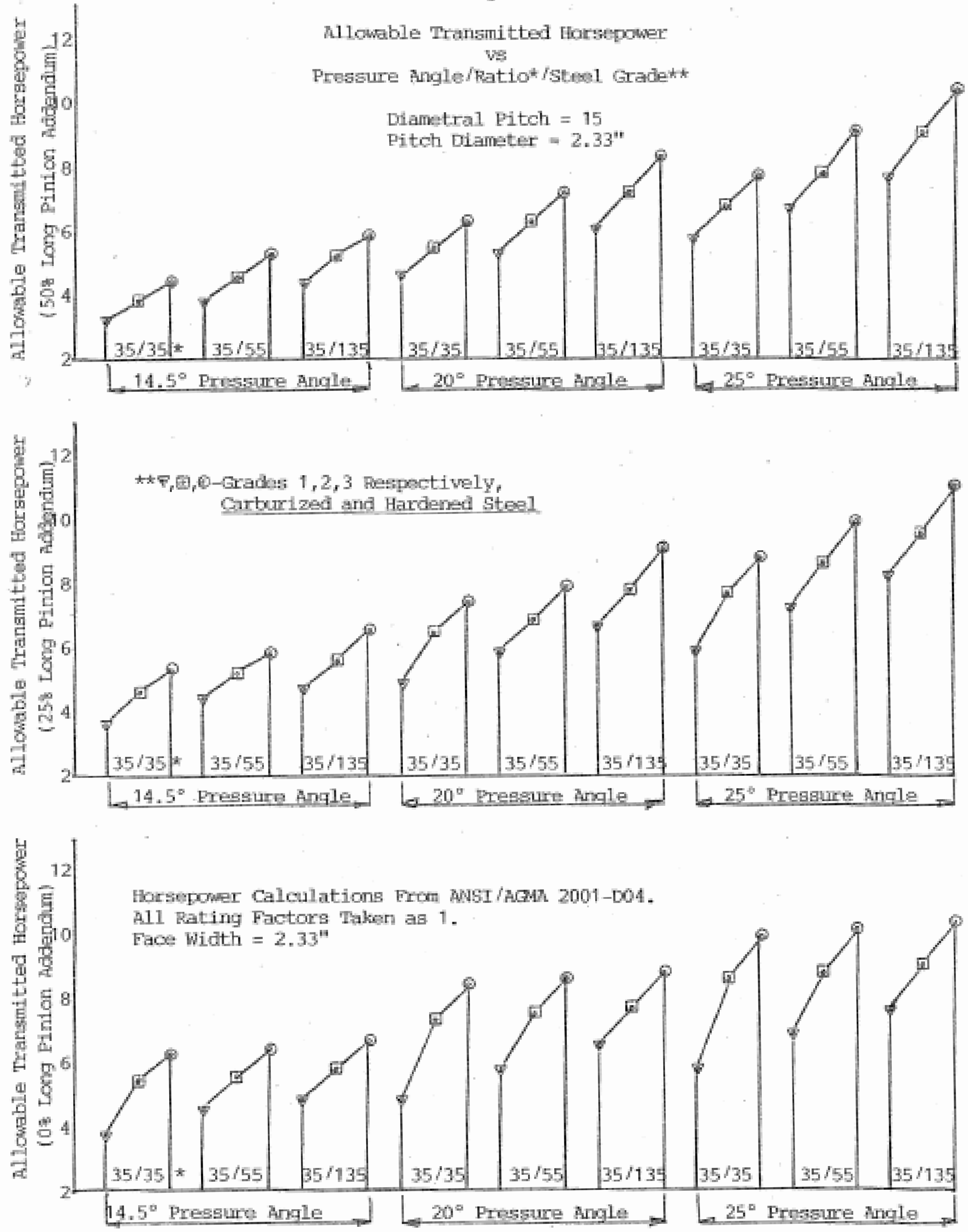


Figure 9
Allowable Transmitted Horsepower
vs
Pressure Angle/Ratio*/Steel Grade**
Diametral Pitch = 5
Pitch Diameter=7.00"
Allowable Transmitted Horsepower (50% Long Pinion Addendum)
230
200
170
140
110
80
35/35
35/55
35/135
14.5° Pressure Angle
20° Pressure Angle
25° Pressure Angle
**∇,□-Grades 1,2 Respectively, Through Hardened Steel
Allowable Transmitted Horsepower (25% Long Pinion Addendum)
Horsepower Calculations From ANSI/AGMA 2001-D04
All Rating Factors Taken as 1.
Face Width = 7.00"
Allowable Transmitted Horsepower (0% Long Pinion Addendum)

Figure 10


Allowable Transmitted Horsepower
vs
Pressure Angle/Ratio*/Steel Grade**
Diametral Pitch = 15
Pitch Diameter = 2.33"
Allowable Transmitted Horsepower (50% Long Pinion Addendum)
**▽,⊡,⊙-Grades 1,2,3 Respectively, Carburized and Hardened Steel
Allowable Transmitted Horsepower (25% Long Pinion Addendum)
Horsepower Calculations From ANSI/AGMA 2001-D04.
All Rating Factors Taken as 1.
Face Width = 2.33"
Allowable Transmitted Horsepower (0% Long Pinion Addendum)
2
4
6
8
10
12
35/35*
35/55
35/135
14.5° Pressure Angle
20° Pressure Angle
25° Pressure Angle

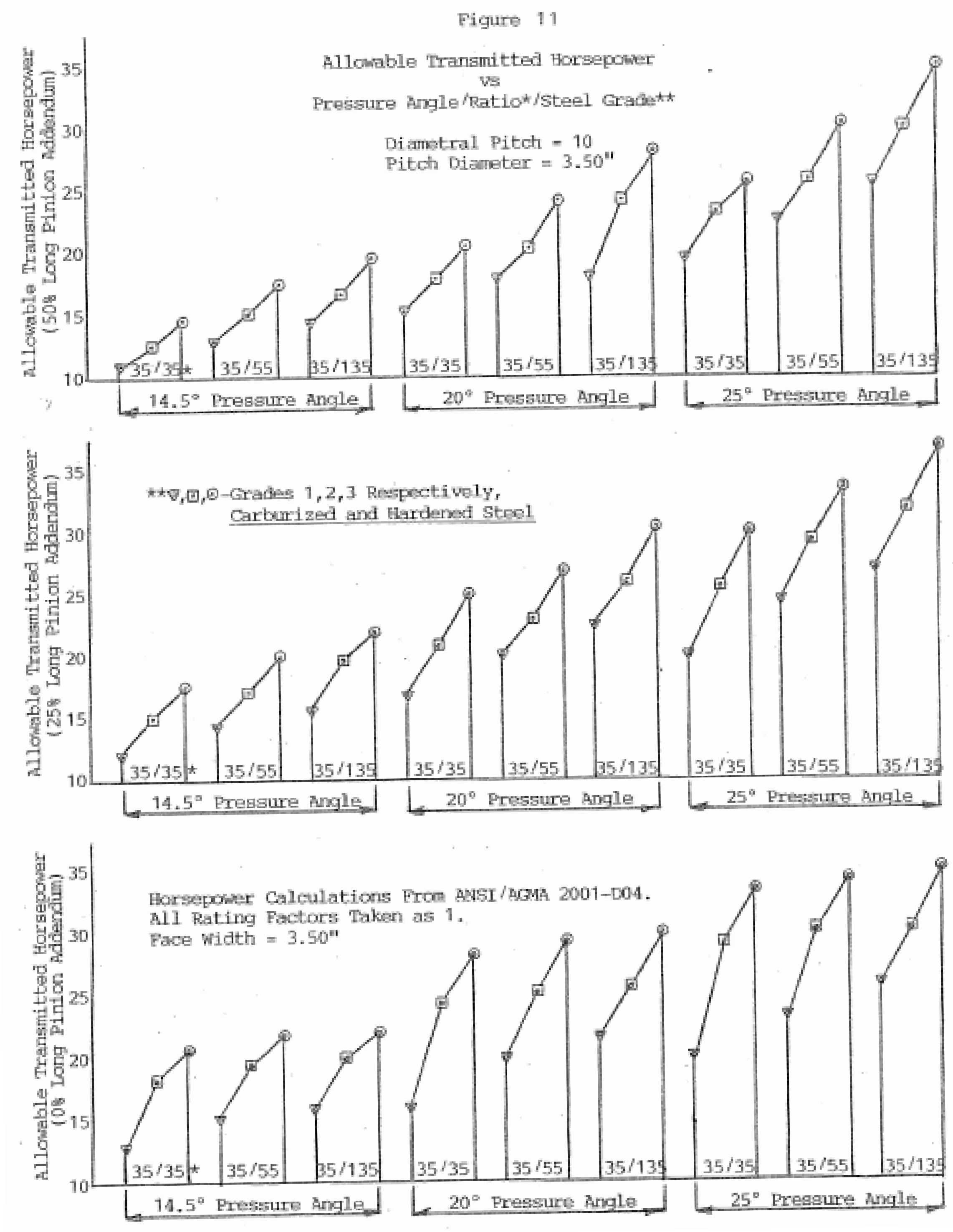


Figure 11
Allowable Transmitted Horsepower
vs
Pressure Angle/Ratio*/Steel Grade**
Diametral Pitch = 10
Pitch Diameter = 3.50"
Allowable Transmitted Horsepower (50% Long Pinion Addendum)
**▽,⊡,⊙-Grades 1,2,3 Respectively,
Carburized and Hardened Steel
Allowable Transmitted Horsepower (25% Long Pinion Addendum)
Horsepower Calculations From ANSI/AGMA 2001-D04.
All Rating Factors Taken as 1.
Face Width = 3.50"
Allowable Transmitted Horsepower (0% Long Pinion Addendum)
35/35*
35/55
35/135
35/35
35/55
35/135
35/35
35/55
35/135
14.5° Pressure Angle
20° Pressure Angle
25° Pressure Angle
10
15
20
25
30
35

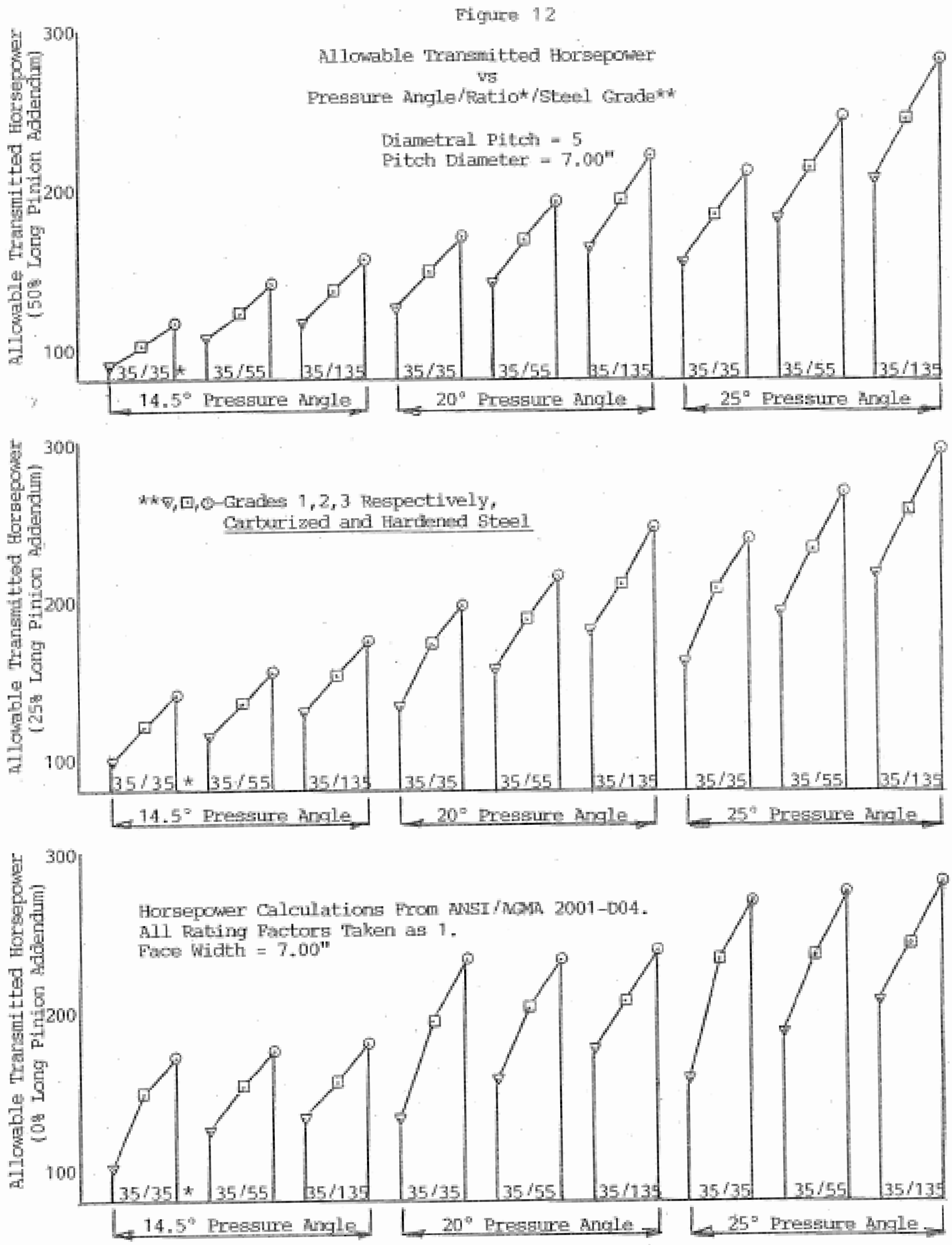


Figure 12
Allowable Transmitted Horsepower
vs
Pressure Angle/Ratio*/Steel Grade**
Diametral Pitch = 5
Pitch Diameter = 7.00"
Allowable Transmitted Horsepower (50% Long Pinion Addendum)
300
200
100
35/35 *
35/55
35/135
14.5° Pressure Angle
20° Pressure Angle
25° Pressure Angle
**▽,⊡,⊙-Grades 1,2,3 Respectively,
Carburized and Hardened Steel
Allowable Transmitted Horsepower (25% Long Pinion Addendum)
Horsepower Calculations From ANSI/AGMA 2001-D04.
All Rating Factors Taken as 1.
Face Width = 7.00"
Allowable Transmitted Horsepower (0% Long Pinion Addendum)

The following conclusions can be drawn from the graphs:

1) Transmitted horsepower increases with pressure angle.

   a) At a pressure angle of 20°, transmitted horsepower increases by about 30% over a pressure angle of 14.5°.

   b) At a pressure angle of 25°, transmitted horsepower increases by about 54% over a pressure angle of 14.5°.

   c) Diametral pitch and ratio have virtually no effect on a and b above.

2) Transmitted horsepower increases with gear ratio.

   a) At a gear ratio of 35/55, transmitted horsepower increases by about 20% over a ratio of 35/35.

   b) At a gear ratio of 35/135, transmitted horsepower increases by about 35% over a ratio of 35/35.

   c) Diametral pitch and pressure angle have virtually no effect on a and b above.

3) The gain in transmitted horsepower as a result of material upgrade is as follows:

   a) There is a 10% gain in horsepower when substituting grade 2 thru-hardened steel for a grade 1 thru-hardened steel when pitting is the failure mode.

   b) There is a 31% gain in horsepower when substituting grade 2 thru-hardened steel for grade 1 thru-hardened steel when bending is the failure mode.

   c) There is a 25% gain in horsepower when substituting grade 2 case-hardened steel for grade 1 case-hardened steel when pitting is the failure mode.

   d) There is an 18% gain in horsepower when substituting grade 2 case-hardened steel for grade 1 case-hardened steel when bending is the failure mode.

e) There is a 53% gain in horsepower when substituting grade 3 case-hardened steel for grade 1 case-hardened steel when pitting is the failure mode.

f) There is a 36% gain in horsepower when substituting grade 3 case-hardened steel for grade 1 case-hardened steel when bending is the failure mode.

g) In general, the failure mode for the data plotted on the graphs was pitting failures for the lower ratios to bending failures for the higher ratios.

4) Heat treatment comparisons are as follows:

a) There is a 14% increase in horsepower when substituting grade 1 case-hardened steel for grade 1 through-hardened steel when pitting is the failure mode.

b) There is a 26% increase in horsepower when substituting grade 1 case-hardened steel for grade 1 through-hardened steel when bending is the failure mode.

c) There is a 29% increase in horsepower when substituting grade 2 case-hardened steel for grade 2 through-hardened steel when pitting is the failure mode.

d) There is a 14% increase in horsepower when substituting grade 2 case-hardened steel for grade 2 through-hardened steel when bending is the failure mode.

e) There is a 58% increase in horsepower when substituting grade 3 case-hardened steel for grade 2 through-hardened steel when pitting is the failure mode.

f) There is a 31% increase in horsepower when substituting grade 3 case-hardened steel for grade 2 through-hardened steel when bending is the failure mode.

g) In general, the failure mode for the data plotted on the graphs was pitting failures for the lower ratios to bending failures for the higher ratios.

5) The previous ratings were made with standard pinion addendum. With 25% long pinion addendum, the following is true:

a) At a pressure angle of 14.5°, there is an average loss of 3.5 horsepower as all three ratios and pitches.

b) At a pressure angle of 20°, the following is true:
   1) There is no gain at a ratio of 35/35.
   2) There is an average gain of 2.5 hp at ratios of 35/55 and 35/155.
   3) The above is true for all three pitches.

c) At a pressure angle of 25°, the following is true:
   1) There is an average gain of 1 horsepower at a ratio of 35/35.
   2) There is an average gain of 2.5 horsepower at a ratio of 35/55.
   3) There is an average gain of 6 horsepower at a ratio of 35/135.
   4) The above is true for all three pitches.

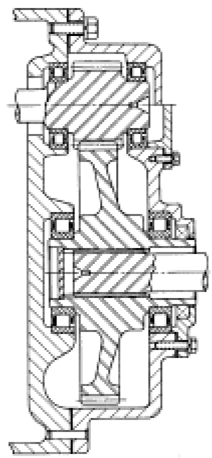
d) With a 50% long pinion addendum, there is a loss of horsepower as all pressure angles, ratios, and pitches.

<u>Spur Gear Application</u>: The drawing at the top of Figure 13 shows a gearbox using spur gears and roller bearings. The pinion is integral with its shaft while the gear is fitted to its shaft. Two different mounting arrangements are shown. One results in better gear alignment due to housing machining improvements.

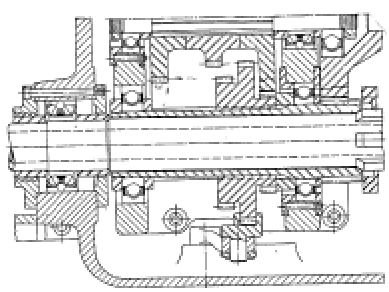
The drawing at the bottom of Figure 13 shows a transmission with spur gears and ball bearings. The three-segment drive gear slides on a splined shaft to engage one of three different output gears to control speed and acceleration. The mounting arrangement allows through-boring of the housing for better gear alignment. It also allows one end of the shaft to be free to accommodate thermal expansion and machining tolerances. Figure 14 has a sketch and formulas for determining gear and bearing loads for automobile transmissions.

<u>Figure 13</u>

<u>Spur gear Application</u>



The housing above the centerline of each shaft has separately machined bearing seats. The design below has through bored bearing seats for better gear alignment.



The main shaft bearing seats are through bored for better gear alignment. The right hand bearing is fixed while the left hand bearing is free to accomodate shaft thermal expansion and machining tolerances.

Figure 14

BEARING LOADS IN AUTOMOBILE TRANSMISSIONS

The determination of bearing sizes for an automobile transmission depends chiefly upon the loads produced during operation in low and intermediate gears. Diagram I illustrates the reactions occurring with second-speed gears in mesh, also the forces which are produced at the tooth contacts of the constant mesh and second-speed gear pairs.

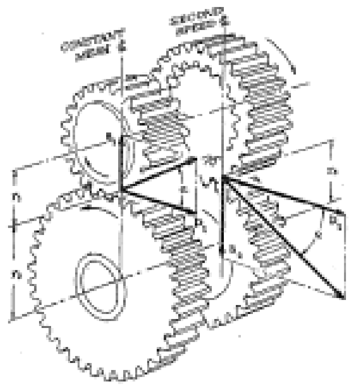


Diagram I.

Q = Maximum engine torque in lbs. inches, taken from torque curve, or estimated on basis of 8 lbs. inches per cubic inch of piston displacement.

r_1 = Pitch radius of constant mesh pinion in inches.

r_2 = Pitch radius of constant mesh gear in inches.

r_3 = Pitch radius of second speed countershaft gear in inches.

α = Pressure angle of teeth.

Then:

$P_1 = \frac{Q}{r_1}$ = Tangential force at pitch circle of constant mesh gears.

$S_1 = P_1 \tan \alpha$ = Separating force at constant mesh gears.

$P_2 = P_1 \frac{r_2}{r_3}$ = Tangential force at pitch circle of second-speed gears.

$S_2 = P_2 \tan \alpha$ = Separating force at second-speed gears.

The effect of these forces on the various transmission bearings is indicated in diagram II, covering second gear operation.

For simplicity, the center lines of the pilot bearing F and the constant mesh pinion have been taken as coincident. As a general rule, this assumption is justified in calculations for conventional gearsets.

Bearing loads, during low and reverse gear operation, may be computed by the same process by substituting the proper tooth forces and reactions based on the respective gear radii and center distances.

Speed Change

r.p.m. of driven gear = r.p.m. of driver x $\frac{\text{Number of teeth in driver}}{\text{Number of teeth in driven}}$

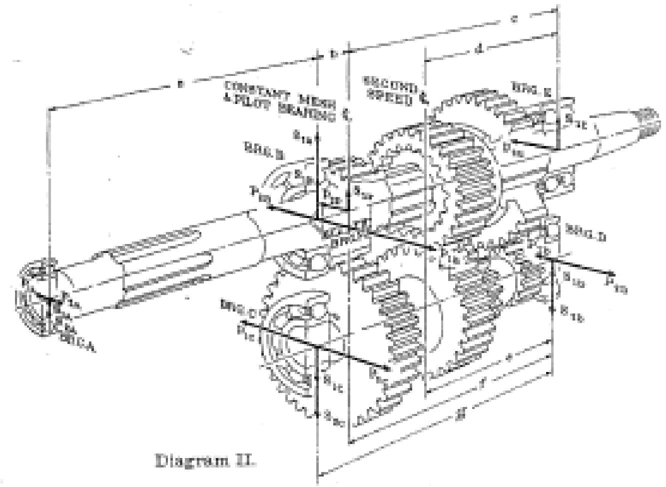


Diagram II.

Computation of Bearing Loads in Second Gear

Due to	on Bearing A	on Bearing B
P_1	$P_1 \frac{b}{a} = P_{1A}$	$P_1 \frac{a+b}{a} = P_1 + P_{1A} = P_{1B}$
S_1	$S_1 \frac{b}{a} = S_{1A}$	$S_1 \frac{a+b}{a} = S_1 + S_{1A} = S_{1B}$
P_2	$P_2 \frac{d}{c} \times \frac{b}{a} = P_{2A}$	$P_2 \frac{d}{c} \times \frac{a+b}{a} = P_{2B'} + P_{2A} = P_{2B}$
S_2	$S_2 \frac{d}{c} \times \frac{b}{a} = S_{2A}$	$S_2 \frac{d}{c} \times \frac{a+b}{a} = S_{2B'} + S_{2A} = S_{2B}$
Total Load	$\sqrt{(P_{1A} - P_{2A})^2 + (S_{1A} + S_{2A})^2}$	$\sqrt{(P_{1B} - P_{2B})^2 + (S_{1B} + S_{2B})^2}$
Due to	on Bearing C	on Bearing D
P_1	$P_1 \frac{f}{g} = P_{1C}$	$P_1 \frac{g-f}{g} = P_1 - P_{1C} = P_{1D}$
S_1	$S_1 \frac{f}{g} = S_{1C}$	$S_1 \frac{g-f}{g} = S_1 - S_{1C} = S_{1D}$
P_2	$P_2 \frac{e}{g} = P_{2C}$	$P_2 \frac{g-e}{g} = P_2 - P_{2C} = P_{2D}$
S_2	$S_2 \frac{e}{g} = S_{2C}$	$S_2 \frac{g-e}{g} = S_2 - S_{2C} = S_{2D}$
Total Load	$\sqrt{(P_{2C} - P_{1C})^2 + (S_{1C} + S_{2C})^2}$	$\sqrt{(P_{2D} - P_{1D})^2 + (S_{1D} + S_{2D})^2}$
Due to	on Bearing E	on Bearing F
P_2	$P_2 \frac{c-d}{c} = P_{2E}$	$P_2 \frac{d}{c} = P_{2F}$
S_2	$S_2 \frac{c-d}{c} = S_{2E}$	$S_2 \frac{d}{c} = S_{2F}$
Total Load	$\sqrt{P_{2E}^2 + S_{2E}^2}$	$\sqrt{P_{2F}^2 + S_{2F}^2}$

Helical Gear Design: The spur gearset that was previously analyzed will be replaced by a helical gearset and the power rating of the two compared. The helix angle of the new gearset will be 10°. All other aspects of the two designs will be the same. As before, grade 1 through-hardened and grade 1 case-hardened steel will be evaluated along with the same four diametral pitches. When analyzing helical gears, the only item that changes in the pitting and bending formulas that were previously used is the tooth geometry factor. Accordingly, a simple tooth geometry factor ratio will be applied to each of the spur gear power ratings to obtain the helical gear ratings.

The helical gear allowable transmitted horsepower based on the pitting resistance of gear teeth contact surfaces is as follows:

1) Thru-hard. steel & 7.00 diam. pitch: 70.9x.255/.132 = 137.0 hp

2) Case-hard. steel & 7.00 diam. pitch: 92.0x.255/.132 = 177.7 hp

3) Thru-hard. steel & 6.77 diam. pitch: 78.1x.255/.132 = 150.9 hp

4) Case-hard. steel & 6.77 diam. pitch: 101.4x.255/.132 = 195.9hp

5) Thru-hard. steel & 6.35 diam. pitch: 95.1x.255/.132 = 183.7 hp

6) Case-hard. steel & 6.35 diam. pitch: 123.4x.255/.132 = 238.4hp

7) Thru-hard. steel & 6.00 diam. pitch: 112.0x.255/.132 = 216.4hp

8) Case-hard. steel & 6.00 diam. pitch: 145.3x.255/.132 = 280.7hp

The helical gear allowable transmitted horsepower based on teeth bending strength is as follows:

1) Thru-hard. steel & 7.00 diam. pitch: 87.7x.56/.30 = 163.7 hp

2) Case-hard. steel & 7.00 diam. pitch: 110.4x.56/.30 = 206.1 hp

3) Thru-hard. steel & 6.77 diam. pitch: 96.7x.56/.30 = 180.5 hp

4) Case-hard. steel & 6.77 diam. pitch: 121.8x.56/.30 = 227.4 hp

5) Thru-hard. steel & 6.35 diam. pitch: 117.6x.56/.30 = 219.5 hp

6) Case-hard. steel & 6.35 diam. pitch: 148.0x.56/.30 = 276.3 hp

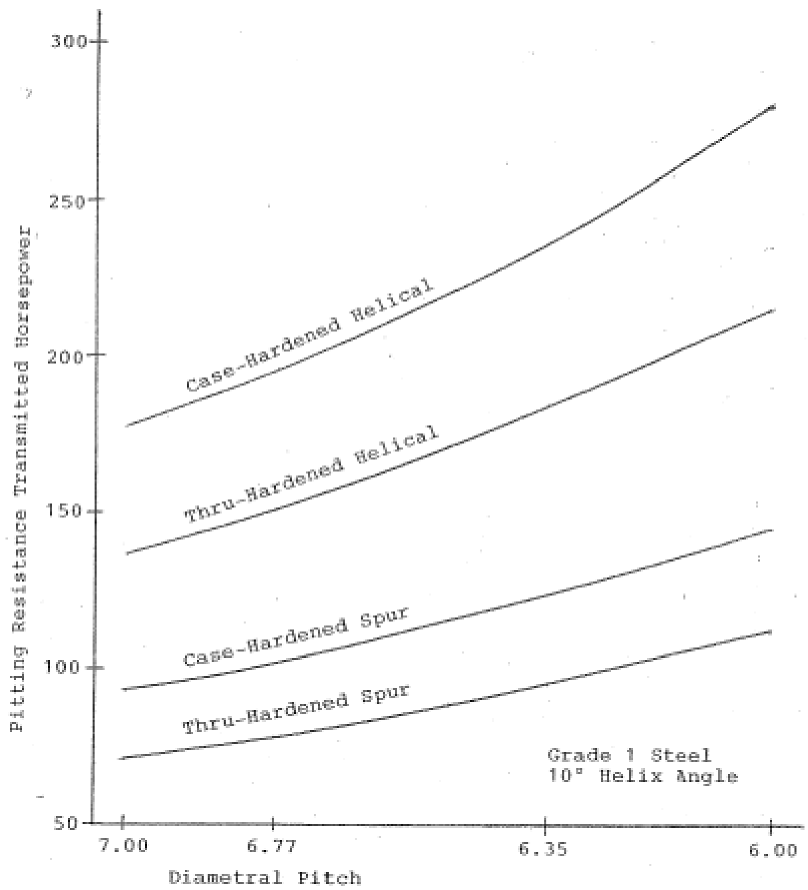
7) Thru-hard. steel & 6.00 diam. pitch: 138.8x.56/.30 = 259.1 hp

8) Case-hard. steel & 6.00 diam. pitch: 174.6x.56/.30 = 325.9 hp

It can be seen from the calculations that the pitting resistance horsepower ratings are still lower than the bending strength horsepower ratings and therefore will be used to compare helical gears to spur gears. Figure 15 shows the comparison with helical gears being almost 2 to 1 better than spur gears. Increasing the helix angle to as much as 30 degrees does not alter the results appreciably.

Figure 15

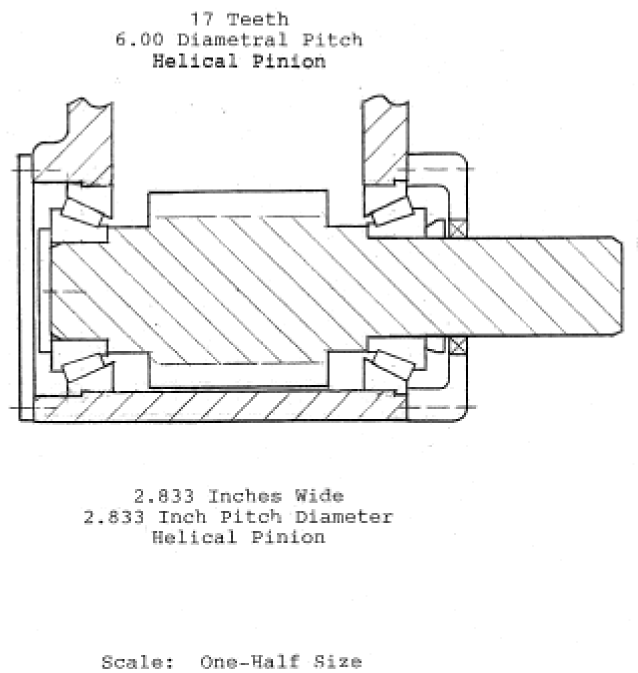
Pitting Resistance Transmitted Horsepower
Versus
Diametral Pitch



Helical Gear Mounting: As another sample problem, the same spur gearset that was previously analyzed will be given helical teeth and reanalyzed. The helix angle will be 10 degrees. The two gears will remain the same size; however, they will be made integral with their respective shafts. This greatly strengthens and stiffens the shafts and counteracts the tendency of helical gears to “tilt” under the overturning moment produced by the thrust load at the mesh. Also, because of this thrust load, roller bearings will be replaced by tapered roller bearings. Unlike straight roller bearings, tapered roller bearings support both radial and thrust loads. The basic size and capacity of the tapered roller bearings will remain the same as the straight roller bearings. See Figure 16.

Figure 16

Helical Gear Mounting



Helical Gear Bearing Selection: Helical gear bearing selection is somewhat more involved than spur gear bearing selection because of the imposition of the thrust load. The thrust load also creates a “thrust couple” that adds to the radial load of one of the shaft bearings and subtracts from the other. Tapered roller bearing analysis itself is also more complicated. A tapered roller bearing on one end of a shaft may impose an “induced thrust” on the bearing at the other end of the shaft. Figure 17 has a sketch and accompanying formulas for calculating the load on the pinion shaft. Disregarding the relative size of the two gears shown on Figure 17, the pinion shaft bearings are number I and II. After solving the equations of Figure 17, the loads on the bearings are as follows:

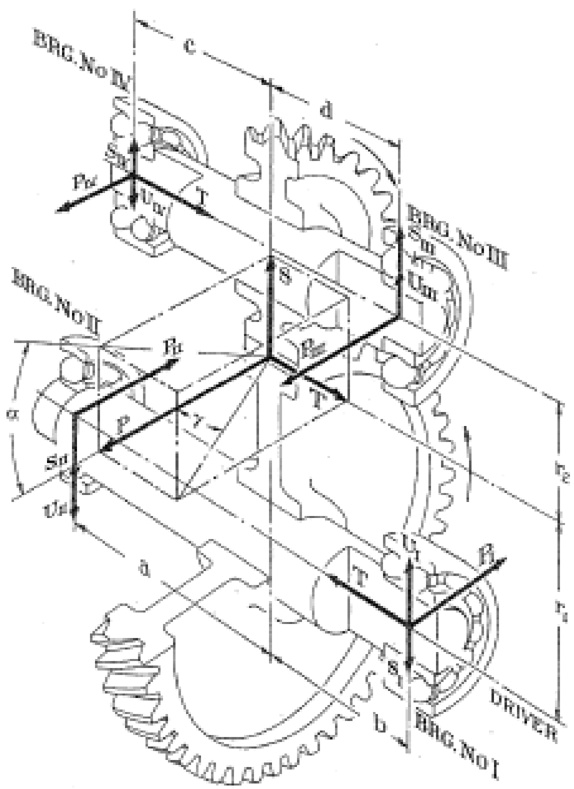
	Radial	Thrust
Bearing I	2383 lb	784 lb
Bearing II	2539 lb	0

Figure 17

Bearing Loads Due to HELICAL SPUR GEARS

In addition to the tangential and separating forces at the tooth contact, helical spur gearing produces a thrust due to the helix angle.

As this thrust is applied at a distance from the axis, a moment, T x r, is set up, which produces additional radial load on the bearings. The thrust is resisted directly by bearings I and IV.



Bearing Loads Due to HELICAL SPUR GEARS

$Q = \frac{H.P. \times 63025}{N}$ = TORQUE INPUT, lbs. inches, where H.P. = horsepower transmitted and N = rev. per min. of driving gear.

$P = \frac{Q}{r_1}$ = TANGENTIAL FORCE, where

r_1 = Pitch radius of driving gear in inches.
[r_2 = Pitch radius of driven gear in inches].

$S = P \tan \alpha$ = SEPARATING FORCE, where

α = Tooth pressure angle measured in plane normal to shaft.

[$S = P \frac{\tan \alpha}{\cos \gamma}$, where α is measured in a plane normal to the tooth, γ being the helix angle defined below].

$\gamma = \tan^{-1} \frac{\pi \times \text{pitch dia.}}{\text{lead}}$ = HELIX ANGLE of tooth, mating gears being of opposite hand.

$T = P \tan \gamma$ = GEAR THRUST, parallel to axis of shaft, due to helix angle, direction depending on hand of helix and rotation.

BEARING LOADS

Due to	on Brg. I	on Brg. II
P	$P\frac{a}{a+b} = P_I$	$P\frac{b}{a+b} = P_{II}$
S	$S\frac{a}{a+b} = S_I$	$S\frac{b}{a+b} = S_{II}$
T	$T\frac{r_1}{a+b} = U_I$	$T\frac{r_1}{a+b} = U_{II} = U_I$
Total Rad. Load	$\sqrt{P_I^2 + (S_I - U_I)^2} = R_I$	$\sqrt{P_{II}^2 + (S_{II} + U_{II})^2}$
Thrust Load	T	
Total Load	$\sqrt{R_I^2 + T^2}$	$\sqrt{P_{II}^2 + (S_{II} + U_{II})^2}$

Due to	on Brg. III	on Brg. IV
P	$P\frac{c}{c+d} = P_{III}$	$P\frac{d}{c+d} = P_{IV}$
S	$S\frac{c}{c+d} = S_{III}$	$S\frac{d}{c+d} = S_{IV}$
T	$T\frac{r_2}{c+d} = U_{III}$	$T\frac{r_2}{c+d} = U_{IV} = U_{III}$
Total Rad. Load	$\sqrt{P_{III}^2 + (S_{III} + U_{III})^2}$	$\sqrt{P_{IV}^2 + (S_{IV} - U_{IV})^2} = R_{IV}$
Thrust Load		T
Total Load	$\sqrt{P_{III}^2 + (S_{III} + U_{III})^2}$	$\sqrt{R_{IV}^2 + T^2}$

Speed Change

r.p.m. of driven gear $= N \times \frac{\text{Number of teeth in driver}}{\text{Number of teeth in driven}}$

With the bearing loads now known, it is necessary to calculate the equivalent radial load for each tapered roller bearing because of the effects of the thrust loading. For pinion bearing number I, the equivalent radial load is derived from three quantities: one from its own radial load, another from the induced thrust caused by bearing II, and the last from the thrust caused by the helical gears. The following formula, similar to the one in the "Bearing Selection Handbook", The Timken Company, is used to calculate bearing I equivalent radial load:

$$P_I = 0.4F_I + K_I\,[(0.47F_{II}/K_{II}) + (T)]$$

P_I is the equivalent radial load in pounds for pinion bearing I.

F_I is the radial load for bearing I previously calculated at 2383 lb.

K_I is a bearing factor for bearing I which is 1.10.

F_{II} is the radial load for bearing II which is 2539 lbs.

K_{II} is the bearing factor for bearing II which is 1.10.

T is the thrust load which was previously calculated at 784 lbs.

Inserting the values and solving the equation, P_I is found to be 3009 lb. For pinion bearing number II, the equivalent radial load is the radial load previously calculated to be 2539 lb. The lives of the two helical gear tapered roller bearings are calculated using the same equation that was previously used for the straight roller bearings:

$$L_I = 3000(3700/3009)^{10/3} \text{ x } (500/1000) = 2{,}987 \text{ B10 hrs}$$

$$L_{II} = 3000(3700/2539)^{10/3} \text{ x } (500/1000) = 5{,}262 \text{ B10 hrs}$$

Dividing by 10 to convert bearing B10 hours to gear 99% reliability hours yields the following:

$$L_I = 2{,}987/10 = 298 \text{ hrs}$$

$$L_{II} = 5{,}262/10 = 526 \text{ hrs}$$

The lives of the two tapered roller bearings are greater than the life of the gear that was previously calculated, so excellent bearing performance can be expected for the application.

Helical Gear Shaft Design: the same ASME "Code for Design of Transmission Shafting" (Mechanical Engineering) formula that was previously used for spur gears will again be used for helical gear shafts. A clause is added to account for axial loads caused by helical gear thrust as shown below:

$$D = [16[(K_m M+aF/8)^2 + (K_t T)^2]^{½} /p_i\, p_t]^{1/3}$$

D is the minimum shaft diameter (in inches) needed to support the bending, torsional, and axial loading.

K_m is a shock rating factor which is 1.5 for rotating shafts.

M is the maximum moment and equals 3682 in-lbs for the pinion shaft. It was calculated using a simply supported beam formula.

a is a stress intensity factor which also will be assumed to be 1.

F is the axial load which is the thrust load of 784 pounds.

K_t is another shock factor which is 1 for rotating shafts.

T is the torque which, as previously calculated, is 6302.5 in-lbs.

p_i is a constant which equals 3.142.

p_t is the allowable shear stress of the shaft which equals 11,340 psi.

Solving the equation for the pinion shaft:

$$D = [16[(3{,}682 + 784/8)^2 + (6302.5)^2]^{½} /3.142 \text{ x } 11{,}340]^{1/3} = 1.562 \text{ in}$$

It can be seen that the calculated shaft diameter of 1.562 inches for the pinion gear is smaller than the stepped shaft smallest diameter of 1.625 inches shown in Figure 16. This example shows how integral shafts can be used to give gears great strength and support when heavier loads are applied.

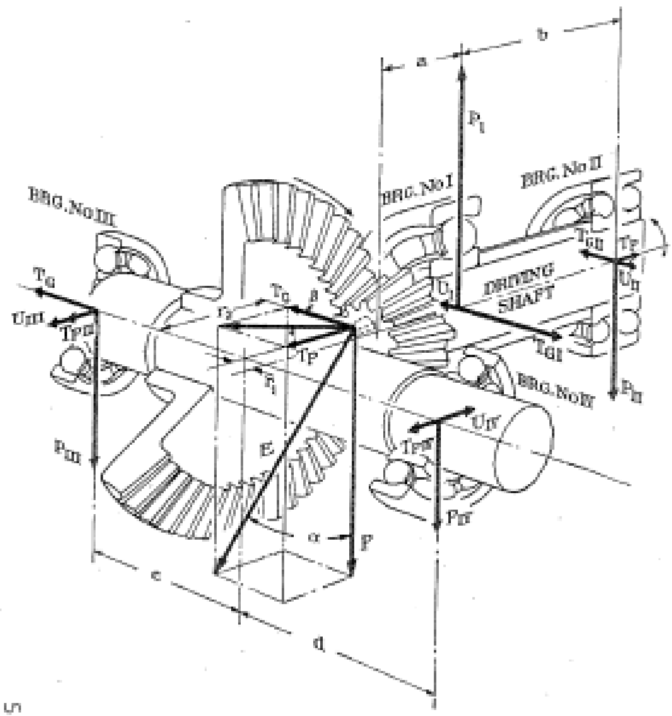
Helical gear shaft misalignment equals .0007 in/in for the pinion. A formula for a simply supported beam was used. Since the tapered roller bearings can tolerate .001 in/in misalignment, shaft rigidity is acceptable for the pinion gear.

Bevel Gears: Bevel gears are normally used to transmit power between two shafts that are 90° apart. In order to provide the proper meshing action, the teeth are tapered as shown on Figure 1. Bevel gears, like spur gears, operate at efficiencies in the high 90% range. Figure 18 has a sketch of a set of bevel gears and formulas to calculate gear and bearing loads.

Figure 18

Bearing Loads Due to PLAIN BEVEL GEARING

In the process of determining bearing loads with this type of gearing, the force E normal to the driving tooth contact is resolved into three forces. The first, P, is directed vertically; the second, T_G, horizontally, both in a plane at right angles to the pinion shaft, and the third, T_P, parallel to the pinion axis.



Bearing Loads Due to PLAIN BEVEL GEARING

$Q = \dfrac{H.P. \times 63025}{N}$ = TORQUE INPUT, lbs. inches, where H.P. = horsepower transmitted and N = revolutions per minute of pinion.

$P = \dfrac{Q}{r_1}$ = TANGENTIAL FORCE, where

r_1 = Mean pinion pitch radius in inches = ½ (pinion pitch diameter — tooth face x sin β), angle β being defined below.

r_2 = Mean gear pitch radius = $r_1 \times \dfrac{\text{Number of teeth in gear}}{\text{Number of teeth in pinion}}$

$T_G = P \tan\alpha \cos\beta$ = GEAR THRUST, where

α = Tooth pressure angle.

β = ½ pinion pitch cone angle.

$= \tan^{-1} \dfrac{\text{Number of teeth in pinion}}{\text{Number of teeth in gear}}$

$T_P = P \tan\alpha \sin\beta$ = PINION THRUST.

BEARING LOADS

Due to	on Brg. I	on Brg. II
P	$P\dfrac{a+b}{b} = P_I$	$P\dfrac{a}{b} = P_{II}$
T_G	$T_G\dfrac{a+b}{b} = T_{GI}$	$T_G\dfrac{a}{b} = T_{GII}$
T_P	$T_P\dfrac{r_1}{b} = U_I$	$T_P\dfrac{r_1}{b} = U_{II} = U_I$
Total Rad. Load	$\sqrt{P_I^2 + (T_{GI} - U_I)^2}$	$\sqrt{P_{II}^2 + (T_{GII} - U_{II})^2} = R_{II}$
Thrust Load		T_P
Total Load	$\sqrt{P_I^2 + (T_{GI} - U_I)^2}$	$\sqrt{R_{II}^2 + T_P^2}$

Due to	on Brg. III	on Brg. IV
P	$P\dfrac{d}{c+d} = P_{III}$	$P\dfrac{c}{c+d} = P_{IV}$
T_G	$T_G\dfrac{r_2}{c+d} = U_{III}$	$T_G\dfrac{r_2}{c+d} = U_{IV} = U_{III}$
T_P	$T_P\dfrac{d}{c+d} = T_{PIII}$	$T_P\dfrac{c}{c+d} = T_{PIV}$
Total Rad. Load	$\sqrt{P_{III}^2 + (T_{PIII} + U_{III})^2} = R_{III}$	$\sqrt{P_{IV}^2 + (U_{IV} - T_{PIV})^2}$
Thrust Load	T_G	
Total Load	$\sqrt{R_{III}^2 + T_G^2}$	$\sqrt{P_{IV}^2 + (U_{IV} - T_{PIV})^2}$

Speed Change

Gear r.p.m. $= N \times \dfrac{\text{Number of teeth in pinion}}{\text{Number of teeth in gear}}$

50

Spiral Bevel Gears: Spiral bevel gears are bevel gears with teeth cut at an angle similar to what a helical gear is to a spur gear. See Figure 1. Spiral bevel gears run more smoothly and are stronger than similar sized bevel gears. Spiral bevel gears operate at lower efficiencies than bevel gears. Figure 19 has a sketch of a spiral bevel gear and formulas to calculate gear and bearing loads.

Figure 19

Bearing Loads Due to

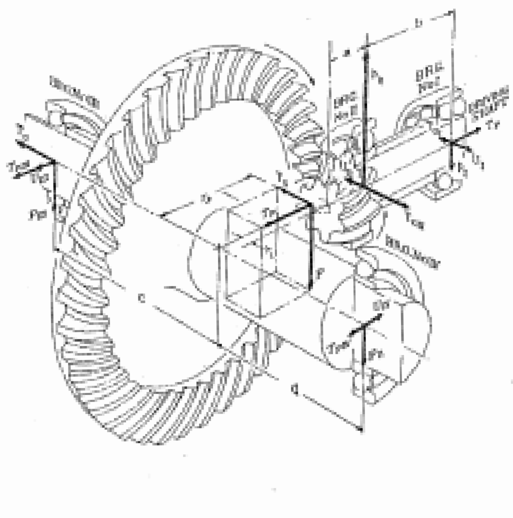
SPIRAL BEVEL GEARING

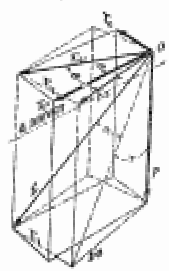
The pinion may rotate clockwise or counter-clockwise (viewed from power input end) and the gears may be cut with left-hand or right-hand spiral. The following combinations are therefore possible:

1. Pinion rotating clockwise with left-hand spiral.
2. Pinion rotating clockwise with right-hand spiral.
3. Pinion rotating counter-clockwise with left-hand spiral.
4. Pinion rotating counter-clockwise with right-hand spiral.

Condition 1 is commonly used, especially for the forward drive in automobile rear axles, in which case reverse drive gives condition 3. The diagram below illustrates condition 1.

Loads are imposed on the bearings by the three components of force E (normal to the driving tooth contact). The first, P, is directed vertically; the second, T_G, horizontally, both being in a plane at right angles to the pinion shaft. The third, T_P, is parallel to the pinion axis. For derivation of these components, see page 44.





Speed Change

$$\text{Gear r.p.m.} = N \times \frac{\text{Number of teeth in pinion}}{\text{Number of teeth in gear}}$$

SPIRAL BEVEL GEARING

$Q = \dfrac{\text{H.P.} \times 63025}{N}$ = TORQUE INPUT, lbs. inches, where H.P. = horsepower transmitted and N = revolutions per minute of pinion.

$P = \dfrac{Q}{r_1}$ = TANGENTIAL FORCE, where

r_1 = Mean pinion pitch radius in inches = ½ (pinion pitch diameter − tooth face x sin β), angle β being defined below.

r_2 = Mean gear pitch radius = $r_1 \times \dfrac{\text{Number of teeth in gear}}{\text{Number of teeth in pinion}}$

α = TOOTH PRESSURE ANGLE.

β = ½ PINION PITCH CONE ANGLE = $\tan^{-1} \dfrac{\text{Number of teeth in pinion}}{\text{Number of teeth in gear}}$

γ = SPIRAL ANGLE.

The values of pinion thrust, T_P, and gear thrust, T_G, are derived from the above data for the four possible combinations as follows:

R.H. SPIRAL / CLOCKWISE	$T_P = P\left(\dfrac{\tan\alpha \sin\beta}{\cos\gamma} - \tan\gamma\cos\beta\right)$
L.H. SPIRAL / C. CLOCKWISE	$T_G = P\left(\dfrac{\tan\alpha \cos\beta}{\cos\gamma} + \tan\gamma\sin\beta\right)$
L.H. SPIRAL / CLOCKWISE	$T_P = P\left(\dfrac{\tan\alpha \sin\beta}{\cos\gamma} + \tan\gamma\cos\beta\right)$
R.H. SPIRAL / C. CLOCKWISE	$T_G = P\left(\dfrac{\tan\alpha \cos\beta}{\cos\gamma} - \tan\gamma\sin\beta\right)$

Note: Positive sign (+) indicates thrust direction away from center. Negative sign (−) indicates thrust direction toward center.

BEARING LOADS

Due to	on Brg. I	on Brg. II
P	$P\,\dfrac{a}{b} = P_I$	$P\,\dfrac{a+b}{b} = P_{II}$
T_G	$T_G\,\dfrac{a}{b} = T_{GI}$	$T_G\,\dfrac{a+b}{b} = T_{GII}$
T_P	$T_P\,\dfrac{r_1}{b} = U_I$	$T_P\,\dfrac{r_1}{b} = U_{II} = U_I$
Total Rad. Load	$\sqrt{P_I^2 + (T_{GI} - U_I)^2} = R_I$	$\sqrt{P_{II}^2 + (T_{GII} - U_{II})^2}$
Thrust Load	T_P	
Total Load	$\sqrt{T_P^2 + R_I^2}$	$\sqrt{P_{II}^2 + (T_{GII} - U_{II})^2}$
Due to	on Brg. III	on Brg. IV
P	$P\,\dfrac{d}{c+d} = P_{III}$	$P\,\dfrac{c}{c+d} = P_{IV}$
T_G	$T_G\,\dfrac{r_2}{c+d} = U_{III}$	$T_G\,\dfrac{r_2}{c+d} = U_{IV} = U_{III}$
T_P	$T_P\,\dfrac{d}{c+d} = T_{PIII}$	$T_P\,\dfrac{c}{c+d} = T_{PIV}$
Total Rad. Load	$\sqrt{P_{III}^2 + (U_{III} + T_{PIII})^2} = R_{III}$	$\sqrt{P_{IV}^2 + (U_{IV} - T_{PIV})^2}$
Thrust Load	T_G	
Total Load	$\sqrt{T_G^2 + R_{III}^2}$	$\sqrt{P_{IV}^2 + (U_{IV} - T_{PIV})^2}$

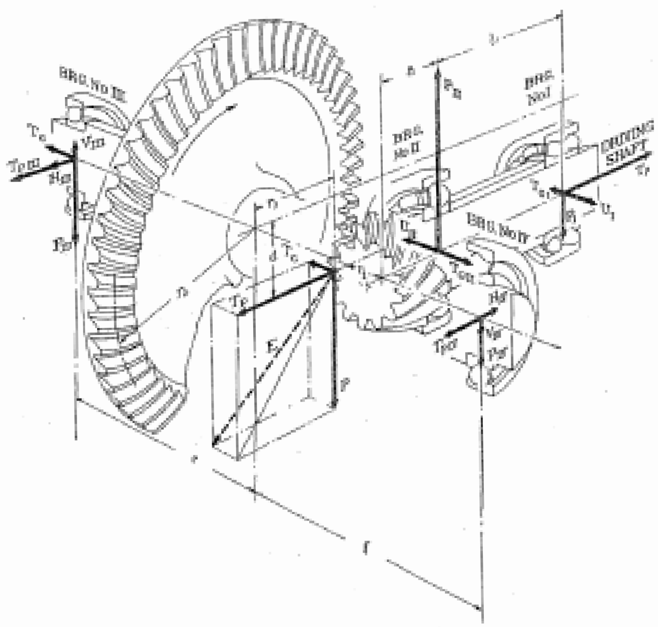
Hypoid Gears: Hypoid gears are like spiral bevel gears except that the axes do not intersect. Hypoid gears are frequently made with spiral angles of 45° or 50°. They are common in drive axles of a wide variety of vehicles. Figure 20 has a sketch of a set of hypoid gears and formulas to calculate gear and bearing loads.

Figure 20

Bearing Loads Due to
HYPOID GEARING

In appearance, hypoid gears are similar to spiral bevel gears; the distinction lies in the fact that while the pinion is being generated by the gear form, it is held in an off-set position, so that the axes of gear and pinion do not intersect. The direction of the off-set determines the hand of the spiral. In the diagram and the calculations based thereon, the left-hand spiral pinion, as commonly applied to automotive rear axles, is selected, with the pinion axis dropped from 1½ to 3½ inches from the horizontal axis of the ring gear.

The tooth action of hypoid gears combines the rolling action of spiral bevel gears with a percentage of endwise sliding. The three actions of the pinion tooth may be derived by the same method as used in ordinary spiral bevel gears.



Speed Change

$$\text{Gear r.p.m.} = N \times \frac{\text{Number of teeth in pinion}}{\text{Number of teeth in gear}}$$

HYPOID GEARING

$Q = \dfrac{H.P. \times 63025}{N}$ = TORQUE INPUT, lbs. inches, where H.P. = horsepower transmitted and N = revolutions per minute.

$P = \dfrac{Q}{r_1}$ = TANGENTIAL FORCE, where

r_1 = Mean pinion pitch radius in inches = ½ (pinion pitch diameter — tooth face x sin β), angle β being defined below.

r_2 = Mean gear pitch radius.

$= r_1 \times \dfrac{\text{Number of teeth in gear}}{\text{Number of teeth in pinion}} \times \dfrac{\cos\text{ (pinion spiral angle)}}{\cos\text{ (gear spiral angle)}}$

α = TOOTH PRESSURE ANGLE on drive side.

β = ½ PINION PITCH CONE ANGLE.

γ = PINION SPIRAL ANGLE.

$T_P = P\left(\dfrac{\tan\alpha \sin\beta}{\cos\gamma} + \tan\gamma\cos\beta\right)$ = PINION THRUST.

$T_G = P\left(\dfrac{\tan\alpha\cos\beta}{\cos\gamma} - \tan\gamma\sin\beta\right)$ = GEAR THRUST.

d = PINION DROP in inches = Distance pinion center line lies below gear center line.

$r_3 = \sqrt{r_2^2 - d^2}$ = EFFECTIVE GEAR RADIUS, or horizontal projection of mean gear radius r_2 defined above.

BEARING LOADS

Due to	on Brg. I	on Brg. II
P	$P\dfrac{a}{b} = P_I$	$P\dfrac{a+b}{b} = P_{II}$
T_P	$T_P\dfrac{r_1}{b} = U_I$	$T_P\dfrac{r_1}{b} = U_{II} = U_I$
T_G	$T_G\dfrac{a}{b} = T_{GI}$	$T_G\dfrac{a+b}{b} = T_{GII}$
Total Rad. Load	$\sqrt{P_I^2 + (U_I - T_{GI})^2} = R_I$	$\sqrt{P_{II}^2 + (U_{II} - T_{GII})^2}$
Thrust Load	T_P	
Total Load	$\sqrt{R_I^2 + T_P^2}$	$\sqrt{P_{II}^2 + (U_{II} - T_{GII})^2}$

Due to	on Brg. III	on Brg. IV
P	$P\dfrac{f}{c+f} = P_{III}$	$P\dfrac{c}{c+f} = P_{IV}$
T_P	$T_P\dfrac{f}{c+f} = T_{PIII}$	$T_P\dfrac{c}{c+f} = T_{PIV}$
T_G	$T_G\dfrac{d}{c+f} = V_{III}$	$T_G\dfrac{d}{c+f} = V_{IV}$
	$T_G\dfrac{r_3}{c+f} = H_{III}$	$T_G\dfrac{r_3}{c+f} = H_{IV} = H_{III}$
Total Rad. Load	$\sqrt{(P_{III} - V_{III})^2 + (T_{PIII} + H_{III})^2} = R_{III}$	$\sqrt{(P_{IV} + V_{IV})^2 + (T_{PIV} - H_{IV})^2}$
Thrust Load	T_G	
Total Load	$\sqrt{T_G^2 + R_{III}^2}$	$\sqrt{(P_{IV} + V_{IV})^2 + (T_{PIV} - H_{IV})^2}$

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